APPLIED GEOPHYSICS

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2018

ELECTRICAL RESISTIVITY TECHNIQUES

Geophysical resistivity techniques are based on the response of the earth to the flow of electrical current. In these methods, an electrical current is passed through the ground and two potential electrodes allow us to record the resultant potential difference between them, giving us a way to measure the electrical impedance of the subsurface material. The apparent resistivity is then a function of the measured impedance (ratio of potential to current) and the geometry of the electrode array. Depending upon the survey geometry, the apparent resistivity data are plotted as 1-D soundings, 1-D profiles, or in 2-D cross-sections in order to look for anomalous regions.

In the shallow subsurface, the presence of water controls much of the conductivity variation. Measurement of resistivity (inverse of conductivity) is, in general, a measure of water saturation and connectivity of pore space. This is because water has a low resistivity and electric current will follow the path of least resistance. **Increasing saturation, increasing salinity of the underground water, increasing porosity of rock (water-filled voids) and increasing number of fractures (water-filled) all tend to *decrease* measured resistivity**. **Increasing compaction of soils or rock units will expel water and effectively increase resistivity**. Air, with naturally high resistivity, results in the opposite response compared to water when filling voids. Whereas the presence of water will reduce resistivity, **the presence of air in voids should increase subsurface resistivity.**

Resistivity measurements are associated with varying depths depending on the separation of the current and potential electrodes in the survey, and can be interpreted in terms of a lithologic and/or geohydrologic model of the subsurface. Data are termed *apparent* resistivity because the resistivity values measured are actually averages over the total current path length but are plotted at one depth point for each potential electrode pair. Two dimensional images of the subsurface apparent resistivity variation are called *pseudo*sections. Data plotted in cross-section is a simplistic representation of actual, complex current flow paths. Computer modeling can help interpret geoelectric data in terms of more accurate earth models.

Geophysical methods are divided into two types : Active and Passive

Passive methods (Natural Sources): Incorporate measurements of natural occurring fields or properties of the earth. Ex. SP, Magnetotelluric (MT), Telluric, Gravity, Magnetic.

Active Methods (Induced Sources) : A signal is injected into the earth and then measure how the earth respond to the signal. Ex. DC. Resistivity, Seismic Refraction, IP, EM, Mise-A-LA-Masse, GPR.

- ***DC Resistivity*** - This is an active method that employs measurements of electrical potential associated with subsurface electrical current flow generated by a DC, or slowly varying AC, source. Factors that affect the measured potential, and thus can be mapped using this method include the presence and quality of pore fluids and clays. Our discussions will focus solely on this method.
- ***Induced Polarization (IP)*** - This is an active method that is commonly done in conjunction with DC Resistivity. It employs measurements of the transient (short-term) variations in potential as the current is initially applied or removed from the ground. It has been observed that when a current is applied to the ground, the ground behaves much like a capicitor, storing some of the applied current as a charge that is dissipated upon removal of the current. In this process, both capacity and electrochemical

effects are responsible. IP is commonly used to detect concentrations of clay and electrically conductive metallic mineral grains.

- ***Self Potential (SP)*** - This is a passive method that employs measurements of naturally occurring electrical potentials commonly associated with the weathering of sulfide ore bodies. Measurable electrical potentials have also been observed in association with ground-water flow and certain biologic processes. The only equipment needed for conducting an SP survey is a high-impedance voltmeter and some means of making good electrical contact to the ground.
- ***Electromagnetic (EM)*** - This is an active method that employs measurements of a time-varying magnetic field generated by induction through current flow within the earth. In this technique, a time-varying magnetic field is generated at the surface of the earth that produces a time-varying electrical current in the earth through induction. A receiver is deployed that compares the magnetic field produced by the current-flow in the earth to that generated at the source. EM is used for locating conductive base-metal deposits, for locating buried pipes and cables, for the detection of unexploded ordinance, and for near-surface geophysical mapping.
- ***Magnetotelluric (MT)*** - This is a passive method that employs measurements of naturally occurring electrical currents, telluric currents, generated by magnetic induction of electrical currents in the ionosphere. This method can be used to determine electrical properties of materials at relatively great depths (down to and including the mantle) inside the Earth. In this technique, a time variation in electrical potential is measured at a base station and at survey stations. Differences in the

recorded signal are used to estimate subsurface distribution of electrical resistivity.

Position of Electrical Methods in:

(1) Petroleum Exploration.

The most prominent applications of electrical techniques in petroleum expl. Are in well logging. Resistivity and SP are standard Logging techniques.

The magnetotelluric method has found important application for pet. Exploration. In structurally complex region (EX. Rocky Mountains).

(2) Engineering & Groundwater.

D C. Resistivity and EM have found broad use in civil Engineering and groundwater studies. Saturated / Unsaturated, Saltwater / freshwater

(3) Mineral Exploration.

Electrical methods interpretation difficult below 1000 to 1500 ft. Electrical exploration methods are the dominant geophysical tools in Mineral Expl.

TABLE 9.1 Applications of Electrical Survey Methods

Classification	Method	Metallic, Nonmetallic Prospecting	Coal, Oil Prospecting, Structural Geology	Engineering Geology, Hydrology
		Electrochemical methods		
Natural field	Self-potential §9.3	Locating sulfide, graphite, pyrite		Locating anomalous water movement, reservoir leakage
Induced field	Induced polarization §9.7	Locating conductors, disseminated sulfides		
		DC methods		
Measuring apparent resistivity	Profiling §9.5	Locating conducting ore bodies and nonconducting veins and tectonic zones	Locating buried faults, dikes, etc.	Locating buried channels, faults, dikes, etc.
	Sounding §9.5	Determining depths of bodies	Determining depths of reference horizons, basement	Determining thickness of overburden, weathered zone, groundwater depth
Measuring equipotentials	Surface electrode §9.5	Locating unpenetrated ore bodies	Locating buried faults, dikes, etc.	Determining dip, anisotropy
	Charged body method (mise-a-la-masse) §9.5	Determining shape, interconnection of penetrated ore bodies		Determining speed of groundwater flow
		Electromagnetic profiling		
Large-low frequency source	Turam method §9.10, 9.11	Locating conducting ores, tectonic zones		Locating aqueous, tectonic zones
Small low-frequency source	Slingram method §9.10	Locating conducting ores, disseminated zones		Locating aqueous, tectonic zones
High-frequency	Radio wave VLF methods	Locating, tracing conducting formations		Locating aqueous, tectonic zones
Transient field	Transient EM profiling §9.10, 9.11	Locating, tracing conducting ores	Locating faults, dikes, tectonic elements	
		Electromagnetic sounding		
Man-made EM field	Frequency sounding §9.10		Investigating vertical changes in resistivity	
Natural Earth field	Telluric current §9.6		Determining thicknesses, resistivity of sediments, layers, basement	
	Magnetotelluric sounding §9.12		Determining resistivities at depth	
Transient field	Transient EM sounding §9.10		Determining resistivities, depths of reference horizons	

Source: After Mares (1984).

Application of Integrated Geophysical Methods

This section indicates which methods are recommended for use on individual deposits, with helpful hints on how to determine which methods one should choose to use on other unlisted deposits. The data are presented in outline form because the "why and how" have been enumerated in previous sections of this manual. A modified version of a cost information graph designed by Geoterex Limited of Canada is included; this information is extremely helpful when planning any geophysical surveys.

Deposits	Direct Detection	Indirect Detection	Geologic Studies, Structures, Etc.
Disseminated sulfide	IP Resistivity SP EM	Magnetics Radiometric (alteration studies) Gravity	Gravity (basin-and-range studies Magnetics (detection of intrustive, etc., helpful in geologic mapping) MT-AMT (basement studies)
Massive sulfide	IP Resistivity Magnetics Gravity SP AMT	Gravity Magnetics	Magnetics VLF, EM, etc.
Uranium	Radiometric studies Radon cups	Radiometric studies Radon cups VLF and ELF IP and resistivity Gas studies (helium and radon) Magnetics	VLF and ELF (structure) Magnetics (geologic mapping) IP and resistivity (rollfront delineation) Magnetics (rollfront studies)
Water	Resistivity EM Seismics AMT	Seismic (aquifer location) Gravity (basement studies)	Magnetics
Asbestos	IP and resistivity Seismics	Magnetics EM	Magnetics EM
Stream channel (placer)		Magnetics (black sand) IP and resistivity (black sand)	Seismic (stream channel location) Resistivity (clay detection)
Lateritic	Seismic soundings Resistivity soundings	Gravity Magnetics	Magnetics (geologic mapping)
Geothermal studies "hot water"	Resistivity Passive seismics EM Geothermal gradient AMT	SP Radiometric studies	Magnetics Gravity
Nondestructive material testing	Dam site evaluation and concrete testing: seismics, ripability studies, etc.; resistivity		
Overburden depth studies	Seismics; resistivity; gravity		
Gravity	Direct detection: seismic studies; resistivity measurements		
Location of pipes or other buried features	Applied potential; magnetics; EM; gravity (detection of voids or caverns)		
Other deposits or objects	General consideration: Determine the physical parameters of the deposit or object and its host. This includes magnetic susceptibility, remnant magnetism, density, resistivity, induced polarizability, color, seismic velocity, radioactivity, temperature, and structural implications as to the location of the deposit or object itself. Choose parameters that would be most likely to differentiate the deposit or object from its host. Run test survey on known area.		

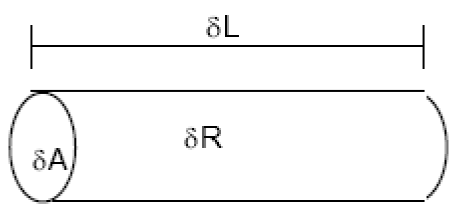
Surface Geophysical Methods in Hydrologic Studies

General Method	Specifc method	Use in study (and depth of exploration)	Type of interpretation (and status)	Limitations	Advantages
ELECTRICAL	**DC Resistivity**	Map complex plumes, complex stratigraphy	Quantitative (production)	Slow; electrical, cultural interference; need larger areas for deep work	Detailed results
	Terrain Conductivity	Map simple plumes and stratigraphy			
	EM 31	(Shallow)	Qualitative (production) quantitative (research)	Electrical, cultural interference	Fast; simple to use
	EM 34-3	(Moderate)	Qualitative (production) quantitative (research)	Electrical, cultural interference	Fast; simple to use
	Time-domain EM 37W	Map plumes and stratigraphy	Qualitative (research)	Complex to use	Small surface area for deep penetration
	Terrain resistivity	Map simple plumes and stratigraphy			
	Very low frequency (VLF) EM 16R	(Moderate)	Qualitative (production) quantitative (research)	Electical, cultural interference	Fast; simple to use
	Ground Penetrating Radar	Map water table and stratigraphy (shallow)	Qualitative (research)	Complex to use	Very fast
	Complex resistivity	Map non-conductive plumes (shallow)	Qualitative (research)	Complex to use; slow; must have clay present	Can map non-conductive contaminant
SEISMIC	**Land refraction**	Map depth to water, depth to rock (shallow or deep)	Quantitative (production)	Cultural noise interference; may need explosives; need larger areas for deep	Detailed results
	Land reflection	Depth to rock (moderate to deep)	Quantitative (research)	Cultural noise interference; may need explosives	Small areas for deep work
	Marine reflection (single	Detailed stratigraphy, depth to rock	Quantitative (research) qualitative	Need water	Very fast

Ohm's Law

Ohm's Law describes the electrical properties of any medium. **Ohm's Law, V = I** R, relates the voltage of a circuit to the product of the current and the resistance. This relationship holds for earth materials as well as simple circuits. **Resistance(R)**, however, is not a material constant. Instead, resistivity is an intrinsic property of the medium describing the resistance of the medium to the flow of electric current.

Resistivity ρ is defined as a unit change in resistance scaled by the ratio of a unit cross-sectional area and a unit length of the material through which the current is passing (Figure 1). **Resistivity** is measured in ohm-m or ohm-ft, and is the reciprocal of the conductivity of the material. Table 1 displays some typical resistivities.



$(\rho = \delta A \, \delta R / \delta L)$

Figure 1. Resistivity is defined based on the change in resistance δR for a given change in length δL and cross-sectional area δA of material.

Table 1

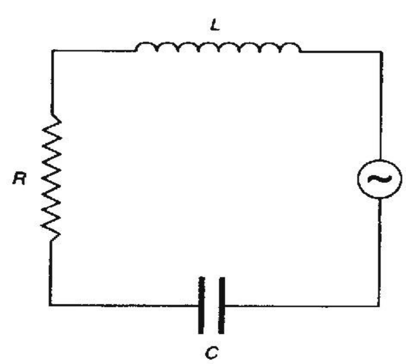
Common Resistivities (ohm-m)

Material Value	Resistivity range	Typical
Igneous & Metamporphic rocks	$10^2 - 10^8$	10^4 10^3
Sedimentary rocks	$10 - 10^8$	10^3
Unconsolidated	$10^{-1} - 10^4$	10^3
Groundwater	1 - 10	5
Pure water		10^3

Note that, in Table 1, the resistivity ranges of different earth materials overlap. Thus, resistivity measurements cannot be directly related to the type of soil or rock in the subsurface without direct sampling or some other geophysical or geotechnical information. Porosity is the major controlling factor for changing resistivity because electricity flows in the near surface by the passage of ions through pore space in the subsurface materials. The porosity (amount of pore space), the permeability (connectivity of pores), the water (or other fluid) content of the pores, and the presence of salts all become contributing factors to changing resistivity. Because most minerals are insulators and rock composition tends to increase resistivity, it is easier to measure conductive anomalies than resistive ones in the subsurface. However, air, with a theoretical infinite resistivity, will produce large resistive anomalies when filling subsurface voids.

<u>Electric circuit has three main properties</u>:

- **Resistance (R):** resistance to movement of charge
- **Capacitance (C):** ability to store charge
- **Inductance (L):** ability to generate current from changing magnetic field arising from moving charges in circuit



<u>Resistance is NOT a fundamental characteristic of the metal in the wire.</u>

MECHANISM OF ELECTRICAL CONDUCTION

Mechanism of electrical conduction in Materials the conduction of electricity through materials can be accomplished by three means :

a) The flow of electrons Ex. In Metal
b) The flow of ions Ex. Salt water .
c) Polarization in which ions or electrons move only a short distance under the influence of an electric field and then stop.

1 Metals :

Conduction by the flow of electrons depends upon the availability of free electrons. If there is a large number of free electrons available, then the material is called a metal, the number of free electrons in a metal is roughly equal to the number of atoms.

The number of conduction electrons is proportional to a factor

$$\mathbf{n \approx \varepsilon^{E/KT}} \qquad \mathbf{E \propto 1/n} \qquad \mathbf{T \propto n}$$

ε : Dielectric constant
K: Boltzman's constant
T: Absolute Temperature.
E Activation Energy.

Metals may be considered a special class of electron semi conductor for which E approaches zero.

Among earth materials native gold and copper are true metals. Most sulfide ore minerals are electron semi conductors with such a low activation energy.

b) The flow of ions, is best exemplified by conduction through water, especially water with appreciable salinity. So that there is an abundance of free ions.

Most earth materials conduct electricity by the motion of ions contained in the water within the pore spaces .

There are three exceptions :

1) The sulfide ores which are electron semi conductors.
2) Completely frozen rock or completely dry rock.
3) Rock with negligible pore spaces (Massive lgneous rooks like gabbro . It also include all rocks at depths greater than a few kilometers, where pore spaces have been closed by high pressure, thus studies involving conductivity of the deep crust and mantle require other mechanisms than ion flow through connate water.

c) Polarization of ions or sometimes electrons under the influence of an electrical field, they move a short distance then stop. Ex. Polarization of the dielectric in a condenser polarization (electrical moment / unit volume)

Conductivity mechanism in non-water-bearing rocks

1) Extrinsic conductivity for low temperatures below 600-750^o k.
2) Intrinsic conductivity for high temperatures.

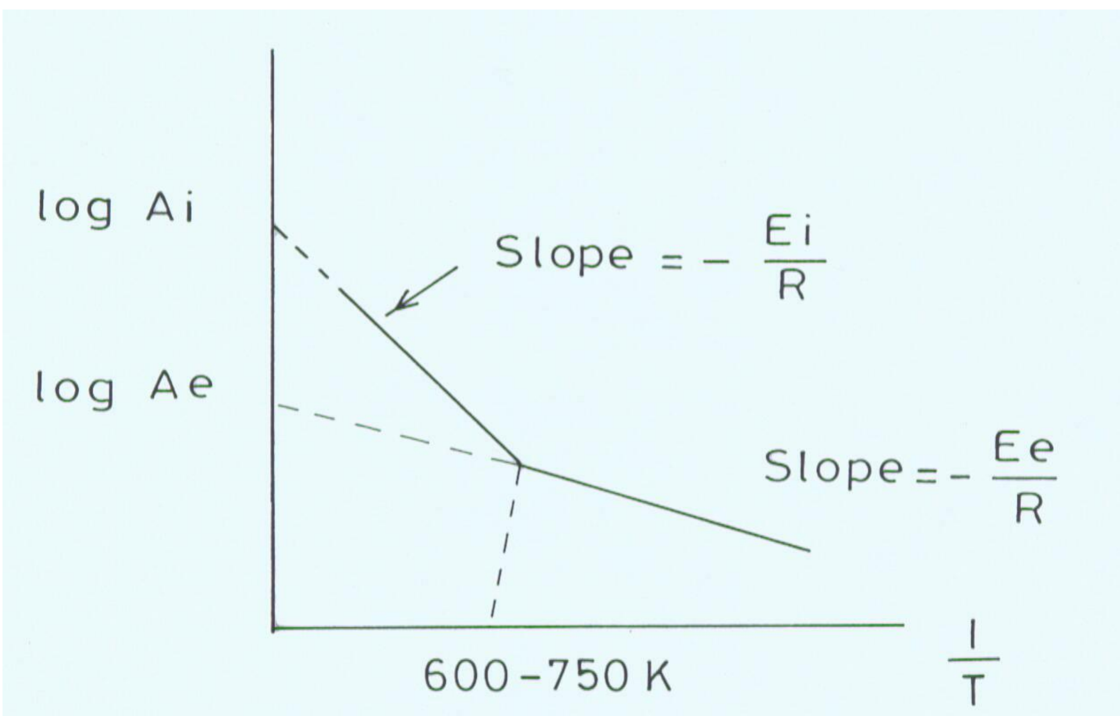
Most electrical exploration will be concerned only with temperatures well below 600-750^o . The extrinsic is due to weakly bonded impurities or defects in the crystal . This is therefore sensitive to the structure of the sample and to its thermal history .

Both of these types of conductivity present the same functional form, hence conductivity vs. temperature for semi conductors can be written :

$$\sigma = Ai\ \varepsilon^{-Ei/RT} + Ae\ \varepsilon^{-Ee/RT}$$

Ai and Ae : Numbers of ions available . Ai is 10^5 times Ae

Ei and Ee are the activation energies . E_i is 2 times as large as E_e .
R: Boltzman's constant



ELECTRICAL PROPERTIES OF ROCKS :

- ☒ Resistivity (or conductivity), which governs the amount of current that passes when a potential difference is created.

- ☒ Electrochemical activity or polarizability, the response of certain minerals to electrolytes in the ground, the bases for SP and IP.

- ☒ Dielectric constant or permittivity. A measure of the capacity of a material to store charge when an electric field is applied . It measure the polarizability of a material in an electric field $= 1 + 4\pi X$
  X : electrical susceptibility .

Electrical methods utilize direct current or Low frequency alternating current to investigate electrical properties of the subsurface.
Electromagnetic methods use alternating electromagnetic field of high frequencies.
Two properties are of primary concern in the Application of electrical methods.

(1) The ability of Rocks to conduct an electrical current.
(2) The polarization which occurs when an electrical current is passed through them (IP).

Resistivity

For a uniform wire or cube, resistance is proportional to length and inversely proportional to cross-sectional area. Resistivity is related to resistance but it not identical to it. The resistance R depends an length, Area and properties of the material which we term resistivity (ohm.m) .

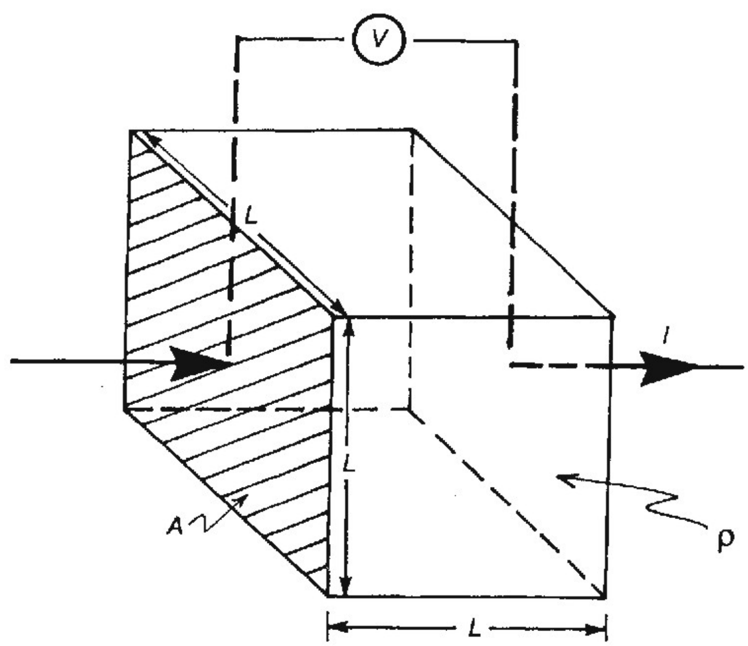
Constant of proportionality is called <u>Resistivity :</u>

$$R = \rho \frac{L}{A}$$

<u>Resistivity is the fundamental physical property of the metal in the wire</u>

$$\rho = \frac{VA}{IL}$$

Resistivity is measured in ohm-m



Conductivity is defined as $1/\rho$ and is measured in Siemens per meter (S/m), equivalent to $ohm^{-1}m^{-1}$.

Copper has a very low resistivity ($1.7 \times 10^{-8}\ \Omega m$) and quartz has a very high resistivity ($1 \times 10^{16}\ \Omega m$). Copper is a CONDUCTOR because of its low resistivity and quartz is an INSULATOR.

We can expect different geologic materials to have greatly different resistivities.

Exercise: In the NEPTUNE project, a cable 1500 km in length might be installed to service observatories. The cable had a copper conductor with a cross section diameter of 0.4 cm. If they send 10 amps down the cable, what will the voltage drop be from shore to the end of the cable?

ρ*(length)* A

$=1.7 \times 10^{-8}\ \Omega m \times 1500000 m/(\pi\ (0.004)^2\ m^2)$

$= \sim 507\Omega$ total cable resistance

V=iR

$=10\ A * 507\Omega = 5.07$ kVolts lost to heating the cable

Anisotropy : is a characteristic of stratified rocks which is generally more conducive in the bedding plane. The anisotropy might be find in a schist (micro anisotropic) or in a large scale as in layered sequence of shale (macro anisotropic) .

Coefficient of anisotropy $\lambda = \rho t / \rho l$

ρl : Longitudinal Resistivety .

ρt : Transverse Resistivity.

The effective Resistivity depends on whether the current is flowing parallel to the layering or perpendicular to it .

$R_{1 = \rho 1\ h1}$

The total Resistance for the unit column (T)

$T = \sum_{\rho 1\ h1}$ Transverse unit resistance

The transverse resistivity ρt is defined by .

$\rho t = T/H$ H is the total thickness

For current flowing horizontally, we have a parallel circuit. The reciprocal resistance is $S = 1/R = \sum hi / \rho i$ Longitudinal unit conductance

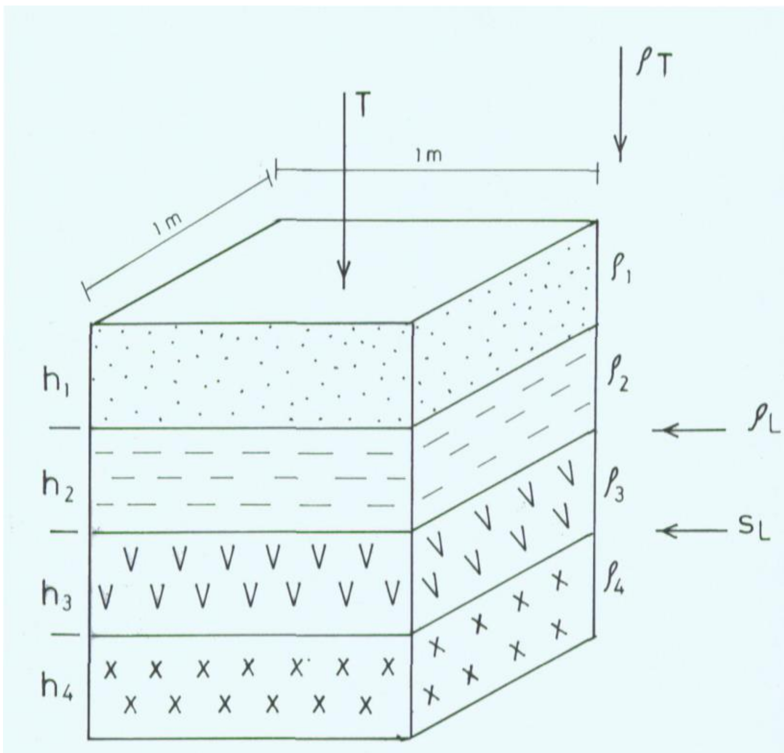
Longitudinal resistivity $\rho_l = H / S$

A geoelectric unit is characterized by two Parameters :

1) Layer Resistivity (ρi)
2) Lager Thickness(ti)

Four electrical parameters can be derived for each layer from the respective resistivity and thickness. There are :

1) Longitudinal conductance $S_L = h/\rho = h.\sigma$
2) Transverse resistance $T = h.\rho$
3) Longitudinal resistivity $\rho l = h/S$
4) Transverse resistivity $\rho t = T/h$



Anisotropy = A = Transverse resistivity ρt / Longitudinal resistivity ρl

The sums of all S_L ($\sum hi / \rho i$) are called Dar Zarrouk functions.
The sums of all T ($\sum hi . \rho i$) are called Dar Zarrouk variables.

Classification of Materials according to Resistivities Values

a) Materials which lack pore spaces will show high resistivity such as
   - massive limestone
   - most igneous and metamorphic (granite, basalt)

b) Materials whose pore space lacks water will show high resistivity such as :
   - dry sand and gravel
   - Ice .

c) Materials whose connate water is clean (free from salinity) will show high resistivity such as :
   - clean sand or gravel , even if water saturated.

d) most other materials will show medium or low resistivity, especially if clay is present such as :
   - clay soil
   - weathered rock.

The presence of clay minerals tends to decrease the Resistivity because :

2) The clay minerals can combine with water .
3) The clay minerals can absorb cations in an exchangeable state on the surface.
4) The clay minerals tend to ionize and contribute to the supply of free ions.

Factors which control the Resistivity

(1) Geologic Age
(2) Salinity.
(3) Free-ion content of the connate water.
(4) Interconnection of the pore spaces (Permeability).
(5) Temperature.
(6) Porosity.
(7) Pressure
(8) Depth

Factors Influencing Electrical Conductivity in Rocks

Porosity (connected/effective - fractures or pores)
Pore saturation (% air or gas)
Hydrocarbon Fluid Saturation
Water salinity (TDS)
Clay Content
Metallic Sulfide Mineral Content
Fluid temperature
Rock Matrix intrinsic resistivity

Archie's Law

Empirical relationship defining bulk resistivity of a saturated porous rock. In sedimentary rocks, resistivity of pore fluid is probably single most important factor controlling resistivity of whole rock.

Archie (1942) developed empirical formula for effective resistivity of rock:

$$\rho_0 = a\rho_w\phi^{-m}$$

ρ_0 = ***bulk rock resistivity***
ρ_w = *pore-water resistivity*
a = empirical constant ($0.6 < a < 1$)

m = *cementation factor* (1.3 poor, unconsolidated) < *m* < 2.2 (good, cemented or crystalline)
φ = *fractional porosity* (vol liq. / vol rock)

Formation Factor*:*

$$F = \frac{\rho_0}{\rho_w} = a\phi^{-m}$$

Effects of Partial Saturation:

$$\rho_t = S_w^{-n} a \rho_w \phi^{-m}$$

Sw is the volumetric saturation.
n is the *saturation coefficient (1.5 < n < 2.5).*

- Archie's Law ignores the effect of pore geometry, but is a reasonable approximation in many sedimentary rocks

Influence of Permeability

A rock with a non-conducting matrix must be permeable (connected pores) as well as porous to conduct electricity.

Darcy's Law:

$$q = -k \frac{dV}{dh},$$

Ohm's Law:

$$j = -\sigma \frac{dV}{dh},$$

q = fluid density,

j = current density,

$\frac{dV}{dh}$ = head or voltage gradient, respectively.

Despite the similarity between Darcy’s and Ohm’s Laws, electric currents have zero
viscosity so even a narrow crack can provide an effective electrical connection between pores that not contribute to hydraulic permeability.

Comparison of electric and hydraulic properties.

Electrical	Hydraulic
Transverse resistance: $T = \Sigma h_i\rho_i = H\rho_l$	Transmissivity: $T_h = \Sigma h_i k_i = K_l H$
Longitudinal conductance: $S = \Sigma h_i/\rho_i = H/\rho_l$	Leakance: $L_h = \Sigma k_i/h_i = K_t/H$
Average aquifer resistivities: ρ_l, ρ_t	Average hydraulic conductivities: K_l, K_t

Field considerations for DC Resistivity

1- Good electrode contact with the earth
   - Wet electrode location.
   - Add Nacl solution or bentonite

2- Surveys should be conducted along a straight line whenever possible .

3- Try to stay away from cultural features whenever possible .
   - Power lines
   - Pipes
   - Ground metal fences
   - Pumps

Sources of Noise

There are a number of sources of noise that can effect our measurements of voltage and current.

1- **Electrode polarization.**
A metallic electrode like a copper or steel rod in contact with an electrolyte groundwater other than a saturated solution of one of its own salt will generate a measurable contact potential. For DC Resistivity, use nonpolarizing electrodes. Copper and copper sulfate solutions are commonly used.

2- **Telluric currents.**
Naturally existing current flow within the earth. By periodically reversing the current from the current electrodes or by employing a slowly varying AC current, the affects of telluric can be cancelled.

3- **Presence of nearby conductors. (Pipes, fences)**
Act as electrical shorts in the system and current will flow along these structures rather than flowing through the earth.

4- **Low resistivity at the near surface.**
If the near surface has a low resistivity, it is difficult to get current to flow more deeply within the earth.

5- **Near- electrode Geology and Topography**
Rugged topography will act to concentrate current flow in valleys and disperse current flow on hills.

6- **Electrical Anisotropy.**
Different resistivity if measured parallel to the bedding plane compared to perpendicular to it .

7- **Instrumental Noise .**

8- **Cultural Feature .**

Applications of Mathematical Methods in Electrical Exploration

Vector Calculus Operations

Three vector calculus operations which find many applications are:

1. The divergence of a vector function

$$\nabla \cdot E = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

2. The curl of a vector function

$$\nabla \times E = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) i + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) j + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) k$$

3. The Gradient of a scalar function

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$$

These examples of vector calculus operations are expressed in Cartesian coordinates.

The Del Operator

The collection of partial derivative operators is commonly called the del operator

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

Gradient ∇f

Divergence $\nabla \cdot E$

Curl $\nabla \times E$

LaPlacian $\nabla \cdot \nabla f = \nabla^2 f$

Vector Identities

In the following identities, u and v are scalar functions while A and B are vector functions. The overbar shows the extent of the operation of the del operator.

$$A \times (B \times C) = (C \times B) \times A = B(A \cdot C) - C(A \cdot B)$$

$$\nabla(uv) = u\nabla v + v\nabla u$$

$$\nabla(A \cdot B) = A \times (\nabla \times B) + (A \cdot \nabla)B + B \times (\nabla \times A) + (B \cdot \nabla)A$$

$$\nabla \cdot uA = u\nabla \cdot A + A \cdot \nabla u$$

$$\nabla \cdot (A \times B) = B \cdot \nabla \times A - A \cdot \nabla \times B$$

$$\nabla \times (uA) = u\nabla \times A - A \times \nabla u$$

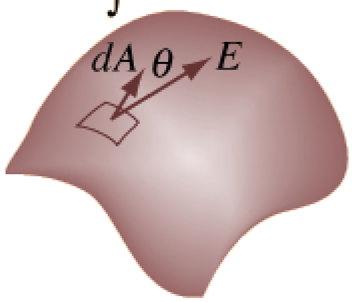
$$\nabla \times (A \times B) = (B \cdot \nabla)A + A(\nabla \cdot B) - (A \cdot \nabla)B - B(\nabla \cdot A)$$

$$(\overline{\nabla \cdot A)B} = (A \cdot \nabla)B + B(\nabla \cdot A)$$

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - (\nabla \cdot \nabla)A$$

Area Integral

$$\int \vec{E} \cdot d\vec{A} = \int E \cos\theta \, dA$$

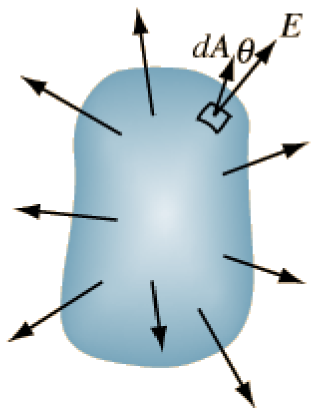


An area integral of a vector function E can be defined as the [integral](#) on a surface of the [scalar product](#) of E with area element dA. The direction of the area element is defined to be perpendicular to the area at that point on the surface.

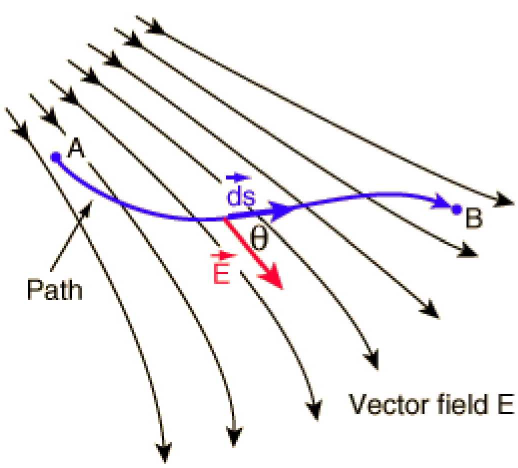
The outward directed surface integral over an entire closed surface is denoted

$$\oint \vec{E} \cdot d\vec{A}$$

It is appropriate for such physical applications as [Gauss' law](#).



Line Integral



Vector functions such as [electric field](#) and [magnetic field](#) occur in physical applications, and [scalar products](#) of these vector functions with another vector such as distance or path length appear with regularity. When such a product is summed over a path length where the magnitudes and directions change, that sum becomes an [integral](#) called a line integral.

$$\int_A^B \vec{E} \cdot d\vec{s} = \int_A^B E \cos\theta \, ds$$

LaPlace's and Poisson's Equations

A useful approach to the calculation of electric potentials is to relate that potential to the charge density which gives rise to it. The electric field is related to the charge density by the divergence relationship

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

E = electric field
ρ = charge density
ε_0 = permittivity

and the electric field is related to the electric potential by a gradient relationship

$$E = -\nabla V$$

Therefore the potential is related to the charge density by Poisson's equation

$$\nabla \cdot \nabla V = \nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

In a charge-free region of space, this becomes LaPlace's equation

$$\nabla^2 V = 0$$

This mathematical operation, the divergence of the gradient of a function, is called the LaPlacian. Expressing the LaPlacian in different coordinate systems to take advantage of the symmetry of a charge distribution helps in the solution for the electric potential V. For example, if the charge distribution has spherical symmetry, you use the LaPlacian in spherical polar coordinates. Since the potential is a scalar function, this approach has advantages over trying to calculate the electric field directly. Once the potential has been calculated, the electric field can be computed by taking the gradient of the potential.

Divergence Theorem

The volume integral of the divergence of a vector function is equal to the integral over the surface of the component normal to the surface.

$$\oint \nabla \cdot E dV = \oint E \cdot dA$$

Stokes' Theorem

The area integral of the curl of a vector function is equal to the line integral of the field around the boundary of the area.

$$\oint \nabla \times E \cdot dA = \oint E \cdot dL$$

The LaPlacian

The divergence of the gradient of a scalar function is called the Laplacian. In rectangular coordinates:

$$\nabla \cdot \nabla f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

The Laplacian finds application in the Schrodinger equation in quantum mechanics. In electrostatics, it is a part of LaPlace's equation and Poisson's equation for relating electric potential to charge density.

Laplacian, Various Coordinates

Compared to the LaPlacian in rectangular coordinates:

$$\nabla \cdot \nabla f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

In cylindrical polar coordinates:

$$\nabla^2 f = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

and in spherical polar coordinates:

$$\nabla^2 f = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{1}{r^2 \sin^2 \theta}\frac{\partial^2 f}{\partial \phi^2} + \frac{2}{r}\frac{\partial f}{\partial r} + \frac{\cot \theta}{r^2}\frac{\partial f}{\partial \theta}$$

Relation of Electric Field to Charge Density

Since electric charge is the source of electric field, the electric field at any point in space can be mathematically related to the charges present. The simplest example is that of an isolated point charge. For multiple point charges, a vector sum of point charge fields is required. If we envision a continuous distribution of charge, then calculus is required and things can become very complex mathematically.

One approach to continuous charge distributions is to define electric flux and make use of Gauss' law to relate the electric field at a surface to the total charge enclosed within the surface. This involves integration of the flux over the surface.

Another approach is to relate derivatives of the electric field to the charge density. This approach can be considered to arise from one of Maxwell's equations and involves the vector calculus operation called the divergence. The divergence of the electric field at a point in space is equal to the charge density divided by the permittivity of space.

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

E = electric field
ρ = charge density
ε_0 = permittivity

In a charge-free region of space where · = 0, we can say

$$\nabla \cdot E = 0$$

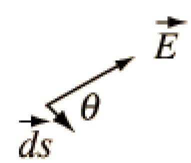
While these relationships could be used to calculate the electric field produced by a given charge distribution, the fact that E is a vector quantity increases the complexity of that calculation. It is often more practical to convert this relationship into one which relates the scalar electric potential to the charge density. This gives Poisson's equation and LaPlace's equation.

Electric Field from Voltage

One of the values of calculating the scalar electric potential (voltage) is that the electric field can be calculated from it. The component of electric field in any direction is the negative of rate of change of the potential in that direction.

If the differential voltage change is calculated along a direction ds, then it is seen to be equal to the electric field component in that direction times the distance ds.

$$dV = -\vec{E} \cdot \vec{ds} = -E_s ds$$



Evaluate the voltage change dV along the direction of ds

The electric field can then be expressed as

$$E_s = -\frac{dV}{ds} \text{ along ds, or } E_s = -\frac{\partial V}{\partial s}$$

This is called a partial derivative.

For rectangular coordinates, the components of the electric field are

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

Express as a gradient.

Electric Field as Gradient

The expression of electric field in terms of voltage can be expressed in the vector form

$$E = iE_x + jE_y + kE_z = -i\frac{\partial V}{\partial x} - j\frac{\partial V}{\partial y} - k\frac{\partial V}{\partial z}$$
$$= -\left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right]V$$

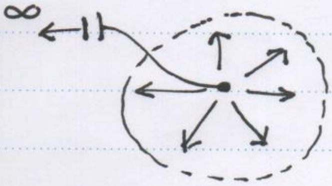
This collection of partial derivatives is called the gradient, and is represented by the symbol ∇ . The electric field can then be written

$$E = -\nabla V$$

Expressions of the gradient in other coordinate systems are often convenient for taking advantage of the symmetry of a given physical problem.

Solution to Laplace's Equation in Electrical Problems

① Field of a single electrode in a whole space



I = total current
r = radial distance
V = electrical potential

$\nabla^2 V = 0$ in spherical coordinates considering only r dependence

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$r^2 \frac{\partial V}{\partial r} = 0$$

$$\frac{\partial V}{\partial r} = \frac{C}{r^2}$$

$$V = \frac{-C}{r} + D$$

To determine C use Ohm's Law $\quad J = \sigma E$

$$\bar{J} = \sigma \nabla V$$

$$J_r = \frac{I}{4\pi r^2} = \sigma \frac{\partial V}{\partial r} = \frac{1}{\rho}\frac{\partial V}{\partial r}$$

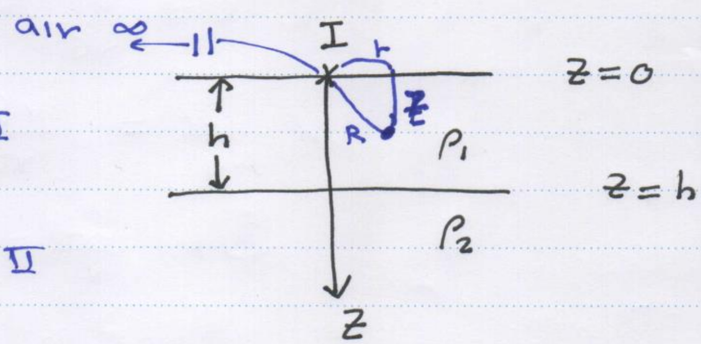
$$\frac{\partial V}{\partial r} = \frac{I\rho}{4\pi r^2} = \frac{C}{r^2} \qquad \therefore\ C = \frac{I\rho}{4\pi}$$

$$\boxed{\therefore\ V = \frac{I\rho}{4\pi r}}$$

2. Single Electrode on a half space

$$V = \frac{I\rho}{2\pi r}$$

3. Single Electrode on One Layer over half space



what is $V(r, z) = ?$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{\partial^2 V}{\partial z^2} = 0$$

Separation of Variables

$$V(r, z) = R(r)\, Z(z)$$

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{Z\, d^2 R}{dr^2} + \frac{Z}{r}\frac{dR}{dr} + R\frac{d^2 Z}{dz^2} = 0$$

$$\frac{1}{R}\frac{d^2 R}{\partial r^2} + \frac{1}{rR}\frac{dR}{dr} + \underbrace{\frac{1}{Z}\frac{d^2 Z}{dz^2}}_{\substack{+k^2 \to e^{-kz} \\ -k^2 \to e^{\pm ikz}}} = 0$$

Since it decays we choose $+k^2$

$$Z \propto e^{\mp kz}$$

where k = eigen Value

$$\frac{d^2 R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + k^2 R = 0 \rightarrow R = J_0(kr)$$

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(K^2 - \frac{n^2}{r^2}\right)R = 0 \rightarrow J_n(kr)$$

Regions I & II

$$V_I(r,z) = \frac{I\rho}{2\pi R} + \int_0^\infty \left\{A(k)e^{-kz} + B(k)e^{kz}\right\} J_0(kr)\,dk$$

$$V_{II}(r,z) = \int_0^\infty C(k)e^{-kz} J_0(kr)\,dk$$

Boundary Conditions (BC's)

Condition 1

1) $J_z\big|_{z=0} = 0 \rightarrow$ No current into air $= \frac{1}{\rho_1}\frac{\partial V_1}{\partial z}\Big|_{z=0} = 0$

2) $V_I = V_{II}$ at boundary $V_I(z=h) = V_{II}(z=h)$

3) $J_z^{I} = J_z^{II}$ at boundary $= \frac{1}{\rho_1}\frac{\partial V_I}{\partial z}\Big|_{z=h} = \frac{1}{\rho_2}\frac{\partial V_{II}}{\partial z}\Big|_{z=h}$

From ohm's Law $J = \sigma \nabla V$

$$J_z = \frac{1}{\rho} \frac{\partial V}{\partial z}$$

From BC I

$$\left.\frac{\partial V_I}{\partial z}\right|_{z=0} = 0 = \frac{I\rho}{2\pi} \frac{-z}{(r^2+z^2)^{3/2}} + \int_0^\infty K \left\{ -A(K) e^{-Kz} + B(K) e^{Kz} \right\} J_0(kr) \Big|_{z=0}$$

$$= \int K(-A+B) J_0 \, dk = 0$$

$$\therefore A = B \quad (1)$$

Weber - Lipschitz Integral

$$\frac{1}{\sqrt{r^2+z^2}} = \int_0^\infty e^{-kz} J_0(kr)\, dk$$

$$V_I(r,z) = \frac{I \rho_1}{2\pi} \int \left\{ (1+A) e^{-kz} + B e^{kz} \right\} J_0(hr)\, dk$$

$$V_{II}(r,z) = \frac{I \rho_2}{2\pi} \int C e^{-kz} J_0(hr)\, dk$$

From B C'' 2

$$\rho_1 \left\{ (1+A) e^{-kh} + B e^{kh} \right\} = \rho_2 C e^{-kh} \quad (2)$$

From B C' 3

$$-(1+A) e^{-kh} + B e^{kh} = -C e^{-kh} \quad (3)$$

From (2) and (3)

$$(2) = (1+A) + B e^{2kh} = \frac{\rho_2}{\rho_1} C$$

③ $-(1+A)+Be^{2kh} = -C$

②/③ $$A = \frac{\rho_2-\rho_1}{(\rho_2+\rho_1)e^{2kh} - (\rho_2-\rho_1)}$$

$$= \frac{S}{e^{2kh}-S} = B$$

where $S = \dfrac{\rho_2-\rho_1}{\rho_2+\rho_1}$ Reflection Coefficient

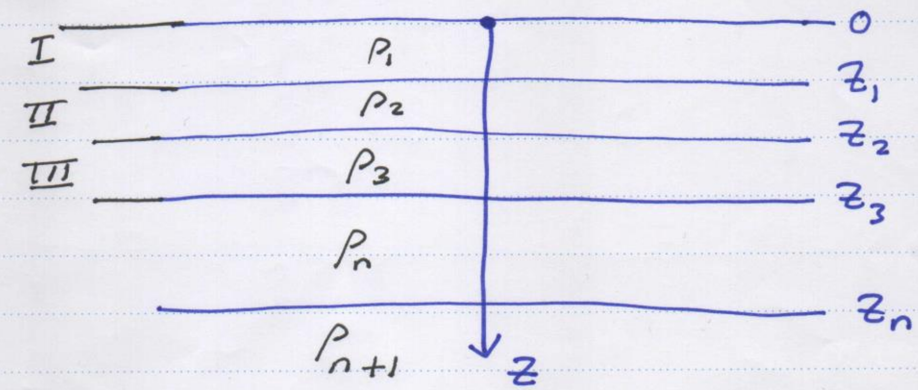
$$V_I(r,z) = \frac{I\rho_1}{2\pi}\left[\frac{1}{R} + \int \frac{S(e^{-kz}+e^{kz})}{e^{2kh}-S} J_0(kr)\,dk\right]$$

at $z=0$

$$V_I(r,z=0) = \frac{I\rho_1}{2\pi}\left[1 + 2SR\int_0^\infty \frac{J_0(kr)}{e^{2kh}-S}dk\right]$$

see Keller & Frishnet integral $= \sum_{n=0}^{\infty} \dfrac{S^{n-1}}{\sqrt{r^2+(2nh)^2}}$

N Layer Case



I) $$V_I = \frac{I\rho_1}{2\pi}\left\{\frac{1}{R} + \int\left[A_1(k)e^{-kz} + B_1(k)e^{kz}\right]J_0(kr)\,dk\right\}$$

$$V_{II} = \frac{I\rho_2}{2\pi}\int\left[A_2(k)e^{-kz} + B_2(k)e^{kz}\right] J_0(kr)\,dK$$

$$V_n = \frac{I\rho_n}{2\pi}\int\left[A_n(k)e^{-kz} + B_n(k)e^{kz}\right]J_0(kr)\,dK$$

$$V_{n+1} = \frac{I \rho_{n+1}}{2\pi} \int A_{n+1}(k) e^{-kz} J_0(kr)\, dk$$

Only we have decay term since $\rho_{n+1} = \frac{1}{2}$ space

We have $2n+1$ unknowns ;
Therefore, we need $2N+1$ BC .

BC's

1) BC at $z=0$ $\left.\frac{\partial V_I}{\partial z}\right|_{z=0} = 0$

2) BC at each Boundary

2N condition
$$\begin{cases} V_i = V_{i+1} \rightarrow \text{Potential is Continuous} \\ \frac{1}{\rho_i}\frac{\partial V_i}{\partial z} = \frac{1}{\rho_{i+1}}\frac{\partial r_{i+1}}{\partial z} \rightarrow \text{Vertical current is Continuous} \end{cases}$$

$$(1+A_1)+B_1 e^{2kz_1} = \frac{\rho_2}{\rho_1}\left(A_2+B_2 e^{2kz_2}\right)$$

$$A_2+B_2 e^{2kz_2} = \frac{\rho_3}{\rho_2}\left(A_3+B_3 e^{2kz_2}\right)$$

$$A_N+B_N e^{2kz_n} = \frac{\rho_{N+1}}{\rho_n} A_{N+1}$$

$$V_0(r, z=0) = \frac{I\rho_1}{2\pi}\int_0^\infty R_N(k) J_0(kr)\, dk$$

Kernel Function $R_N(k) = \dfrac{1-Q_N e^{-2k(z_2-z_1)}}{1+Q_N e^{-2k(z_2-z_1)}}$

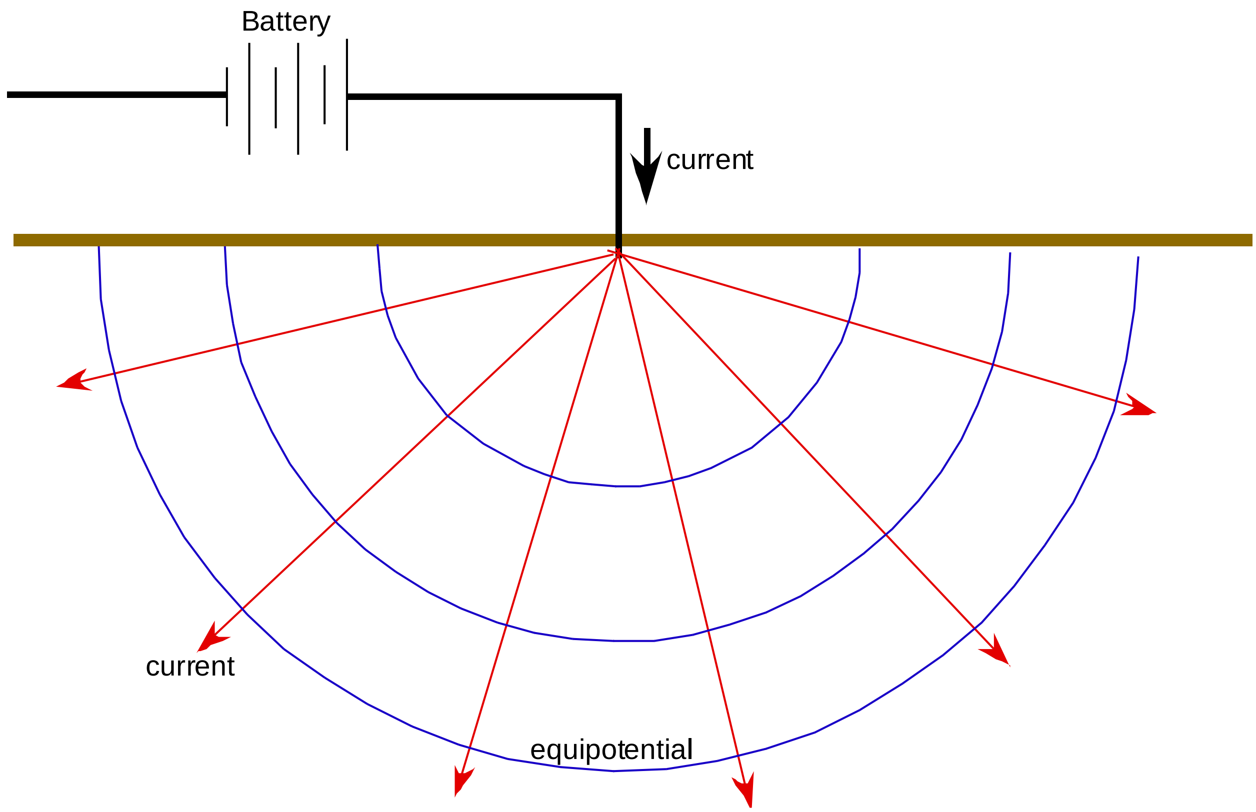
$$Q_N = \frac{\rho_1-\rho_2 R_{N-1}}{\rho_1+\rho_2 R_{N-1}}$$

Finally $R_1 = \dfrac{1-Q_1 e^{-2k(z_n-z_{n-1})}}{1+Q_1 e^{-2k(z_n-z_{n-1})}}$

$Q_1 = \dfrac{\rho_1-\rho_2}{\rho_1+\rho_2}$ where $R_0=1$

<u>Current Flow in Uniform Earth</u>

As is the case for gravity and magnetics, we will find that electrical **potential**, measured in Volts, has the same properties as gravity and magnetic potentials, in that it is a scalar, and we can add the effects of different sources of potential to find out where current will flow. <u>Current will flow in a direction normal to equipotential (equal voltage) surfaces.</u>
Rather than have current flow only through wires, we will now plug our wires into the earth and see how current flows through the earth, and how to measure it to determine regions of anomalous resistivity.
Consider an electrode stuck in the ground with it's matching electrode far away (just like a magnetic monopole). It's potential relative to the distant electrode is measured in Volts.



If we measure the potential difference between two shells at some a distance D from the electrode, we get

$$dV = iR = i\left(\rho\frac{l}{A}\right) = i\left(\rho\frac{dr}{2\pi r^2}\right)$$

where dr is the thickness of the shell across which we measure the potential, Recalling that the resistivity of air is so high, no current will flow through it, so we only need have the surface of a hemisphere ($2\pi r^2$).

We now integrate in from infinity (where potential is zero) to get the potential at a point a distance D from the source:

$$V = \int_D^\infty dV = \frac{i\rho}{2\pi}\int_D^\infty \frac{dr}{r^2} = \frac{i\rho}{2\pi D}$$

IF the resistivity of the ground is UNIFORM.

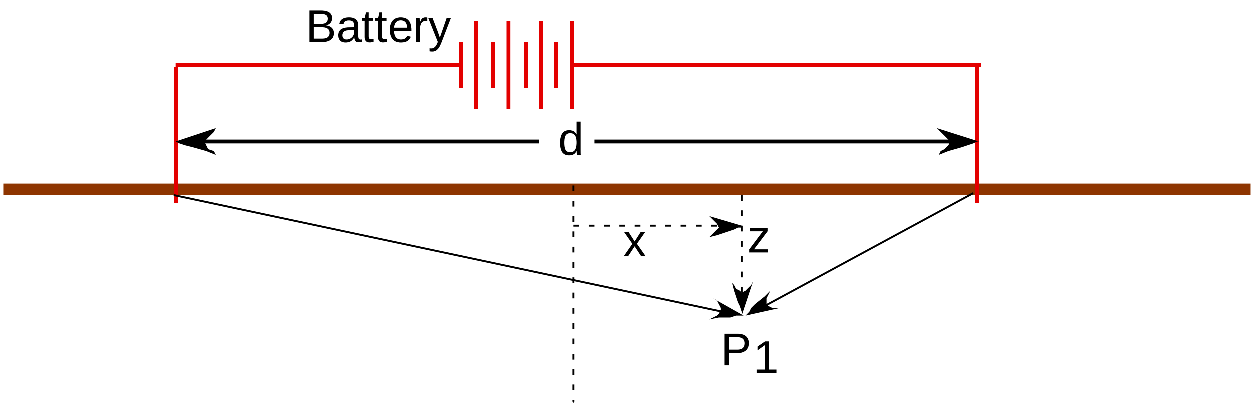
The current, *i*, above is the current IN THE WIRE, not the current in the ground, which varies.

This is the <u>basic equation of resistivity</u>, in that we can add the potentials from many sources to obtain a "potential" map of a surface. By contouring that map, we have <u>equipotential</u> lines, along which no current flows. Current flows in directions perpendicular to equipotential lines.

Sound familiar? It should! Magnetic lines of force are perpendicular to magnetic equipotential surfaces, and the pull of gravity is perpendicular to gravity equipotential surfaces.

TWO ELECTRODES:

What if we move the other current electrode in from far away?



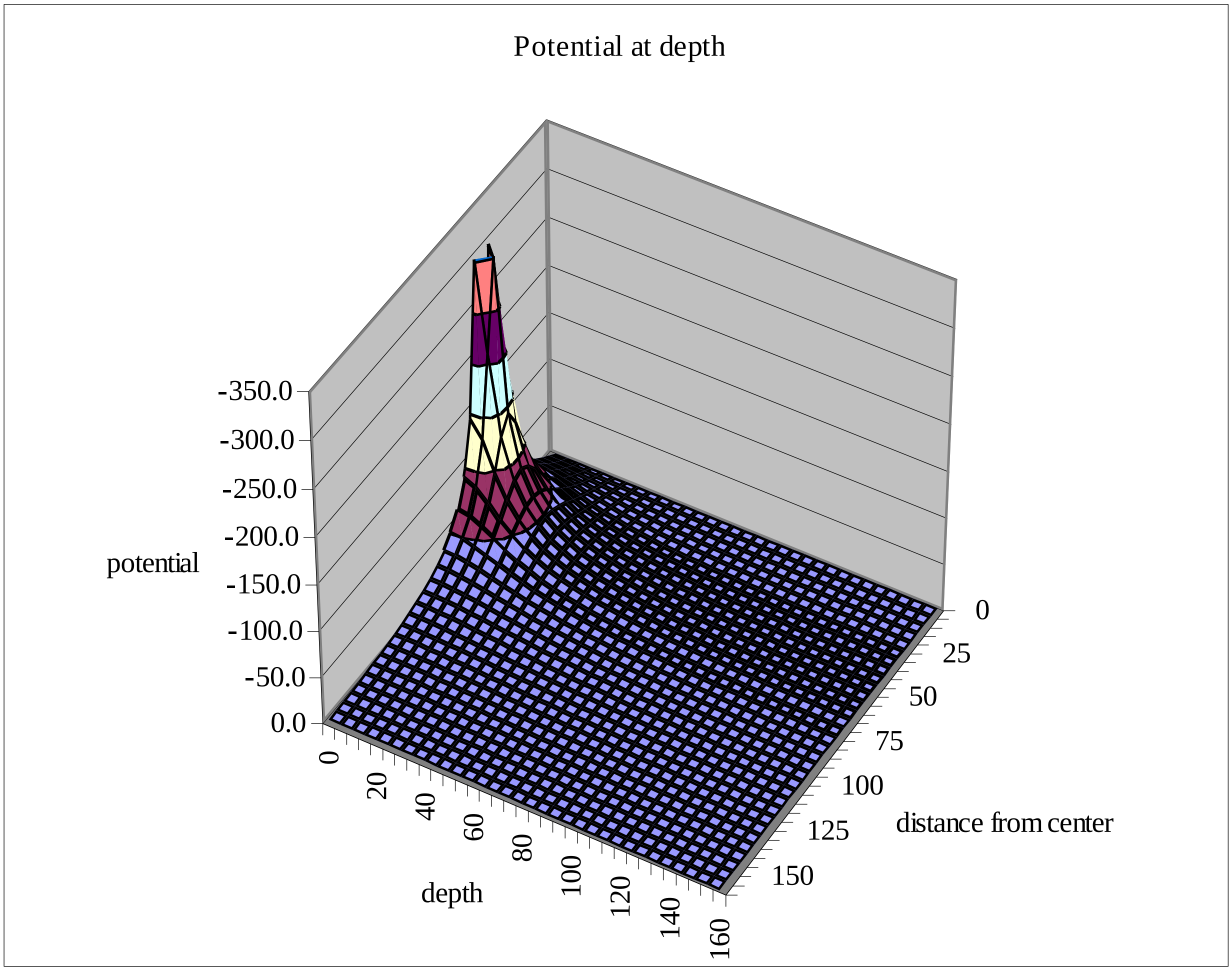
We can calculate the potential at point P_1 by just adding the potentials from both current electrodes - remembering that one is positive, and the other negative:

$$V_{P_1} = \frac{i\rho}{2\pi r_1} - \frac{i\rho}{2\pi r_2} = \frac{i\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

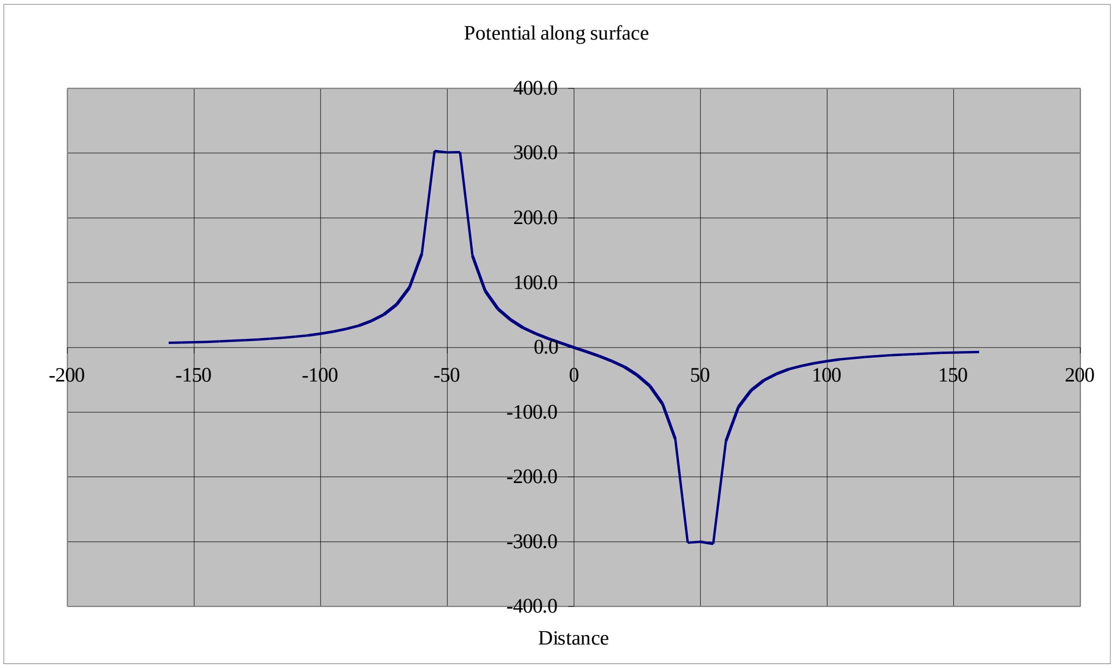
What is the potential between the electrodes vs. depth?

We just change the r values to x-z coordinates:

$$V_{P_1} = \frac{i\rho}{2\pi}\left\{\frac{1}{\sqrt{\left(d/2 + x\right)^2 + z^2}} - \frac{1}{\sqrt{\left(d/2 - x\right)^2 + z^2}}\right\}$$



We can’t measure the potential below the surface in the field, though. We’re stuck at the surface. The surface potential looks like the profile below for a current of 1 A, 100 m between electrodes, and a resistivity of 10kΩm



If we put voltage probes at –65m and +65m along the x axis above, what voltage would we see?

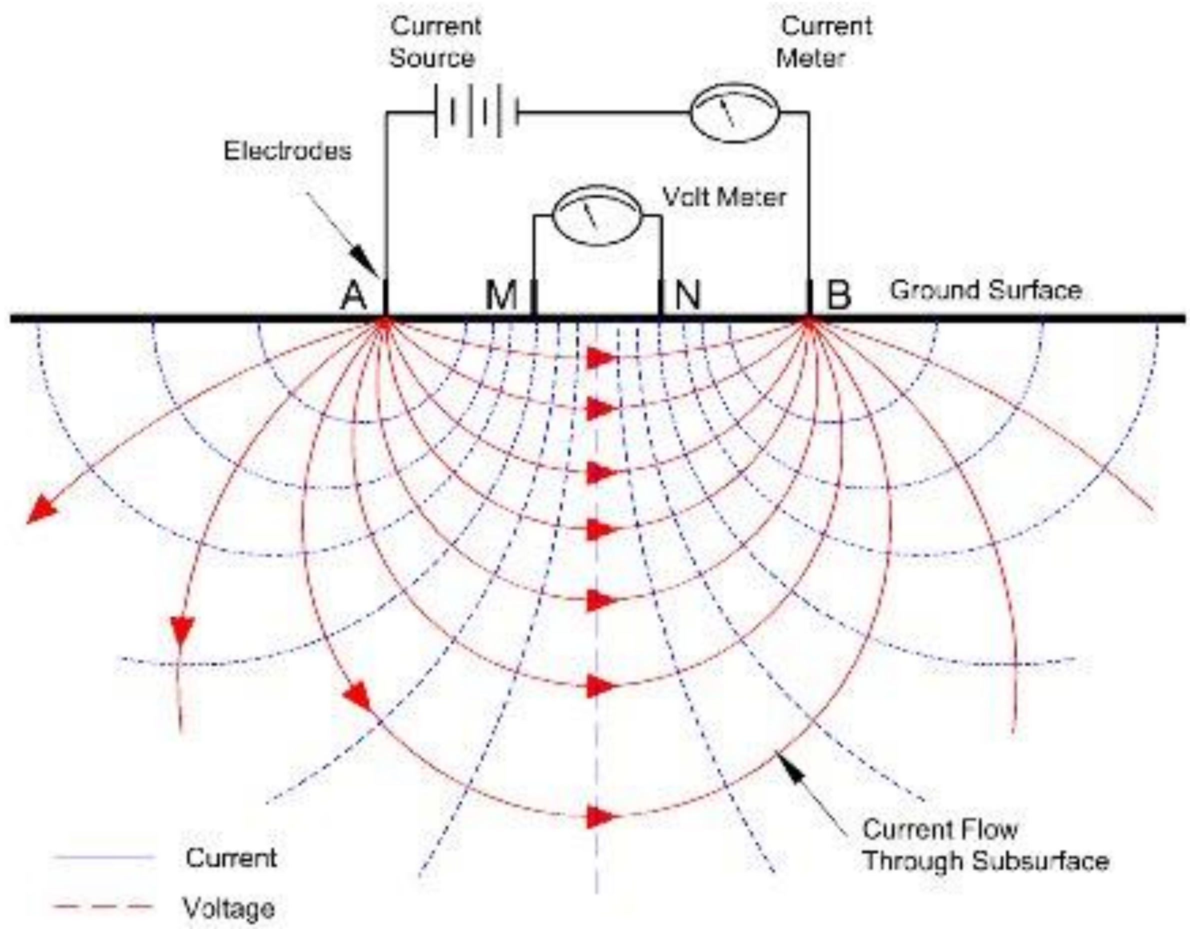
This allows us to contour equipotential lines, but how much current is flowing in what areas? Current flows ALL THROUGH the subsurface, not just directly from one electrode to the other. With some difficulty, it can be shown that the fraction of the total current (i_f) flowing above a depth z for an electrode separation d is given by:

$$i_f = \frac{2}{\pi} \tan^{-1} \frac{2z}{d}$$

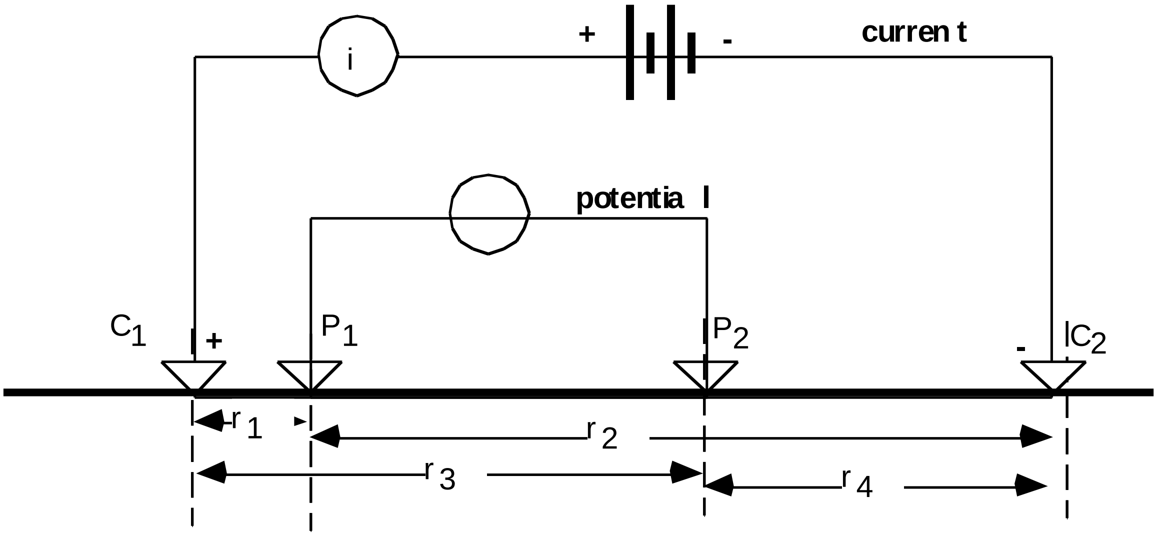
In a region of equal resistivity - about 70% of the current flows at depths shallower than the distance between the electrodes.

With this information, we can sketch lines perpendicular to the equipotentials that show where most of the current is flowing. Be careful, though, **this only works where the resistivity is constant throughout the model**! Note that all current lines are perpendicular to equipotential lines - no current flows between two points where the potential is equal.

The most common form of resistivity measurement uses two current electrodes and two potential electrodes:



We use the same argument, summing potentials, to obtain the voltage across two electrodes we get:



The potential difference between P1 and P_2 is:

$$V_{P_1-P_2} = V_{P_1} - V_{P_2}$$

$$V_{P_1} = \left(V_{C_1} + V_{C_2}\right)\Big|_{P_1}, \quad V_{P_2} = \left(V_{C_1} + V_{C_2}\right)\Big|_{P_2}$$

$$V_{P_1-P_2} = \left(\frac{i\rho}{2\pi r_1} - \frac{i\rho}{2\pi r_2}\right) - \left(\frac{i\rho}{2\pi r_3} - \frac{i\rho}{2\pi r_4}\right) = \frac{i\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}\right)$$

Solving for the resistivity,:

$$\rho = \frac{2\pi V_{P_1-P_2}}{i} \frac{1}{\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}}$$

Thus, we can measure the current, voltage, and appropriate distances and solve for resistivity.

In the example above we have current electrodes at ±50m, and voltage electrodes at ± 65m, so:

r_1= 15m

r_2= 115

r_3= 115

r_4= 15

and the current is 1.0 A

So we can solve for ρ=(2 π*185 /1)*1/(1/15-1/115-1/115+1/15)= 1025.6 Ωm.

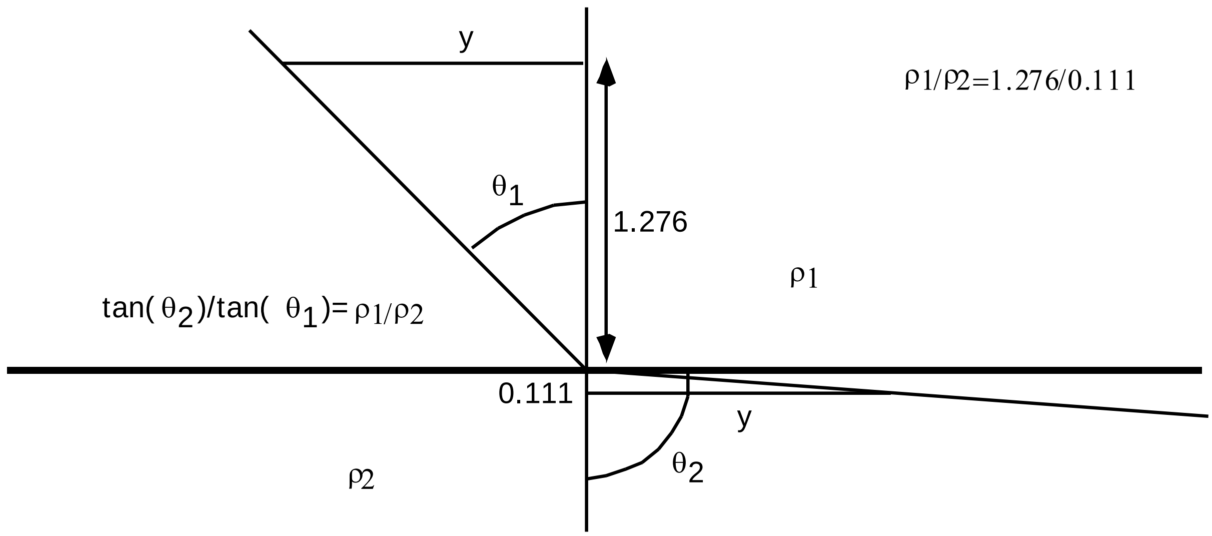
Which is pretty close to the model value of 135 Ωm.

BUT, this is a boring model; what we really want to know is what to expect as the resistivity changes with depth.

CURRENT DENSITY AND FLOW LINES

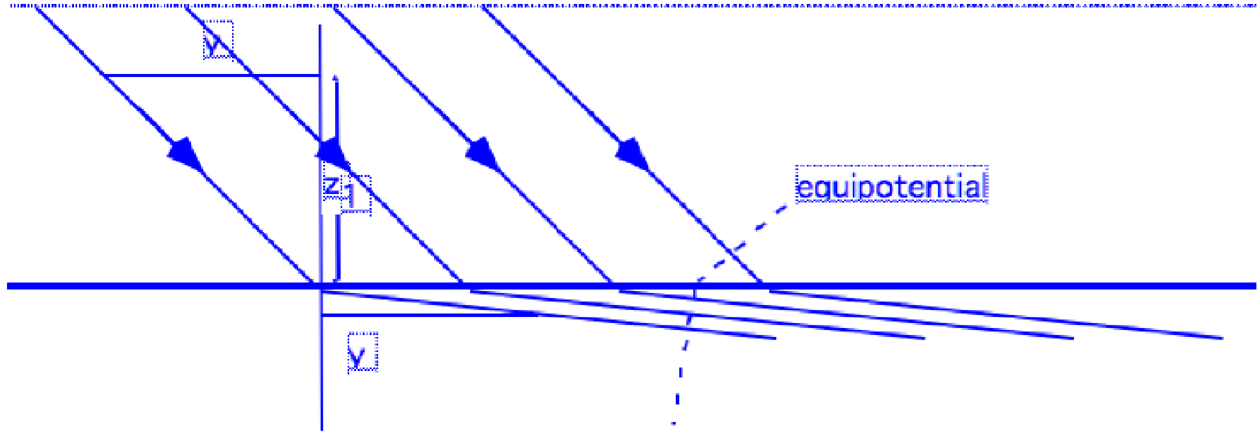
If we think about current flow lines crossing the boundary between two resistivities, it's almost like a seismic ray passing between two materials with different velocities - but the formula is different:

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\rho_2}{\rho_1}$$



Note that this is equivalent to $z_1/z_2=r_1/r_2$, where z is the distance along the vertical axis. So, if we make z_i proportional to r_i then z_2 is proportional to r_2, holding y constant, then we will get the current flow direction easily.

Note that he current flow lines get closer together when the current moves into a region of lower reisistivity:



implying that the current density increases as we cross to the lower resistivity material. If resistivity increases with depth, then current density decreases. If resistivity in a region is VERY high (insulator), then few flow lines will cross a boundary with a conductor, and those that do will be directed perpendicular to the boundary.

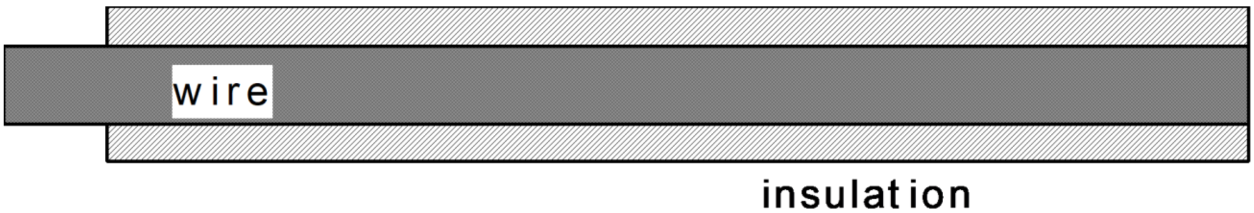
We can define the APPARENT RESISTIVITY as the resistivity we would get assuming that no boundary or change in resistivity is present. So that apparent resistivity equation is identical to the equation for a material with constant resistivity:

$$\rho_a = \frac{2\pi\Delta V_{P_1-P_2}}{i}\left(\frac{1}{\frac{1}{r_1}-\frac{1}{r_2}-\frac{1}{r_3}+\frac{1}{r_4}}\right)$$

So, what does this tell us?
Recall that current density is qualitatively measured by the number of flow lines. What is the relationship between potential difference ΔV and current density j ?

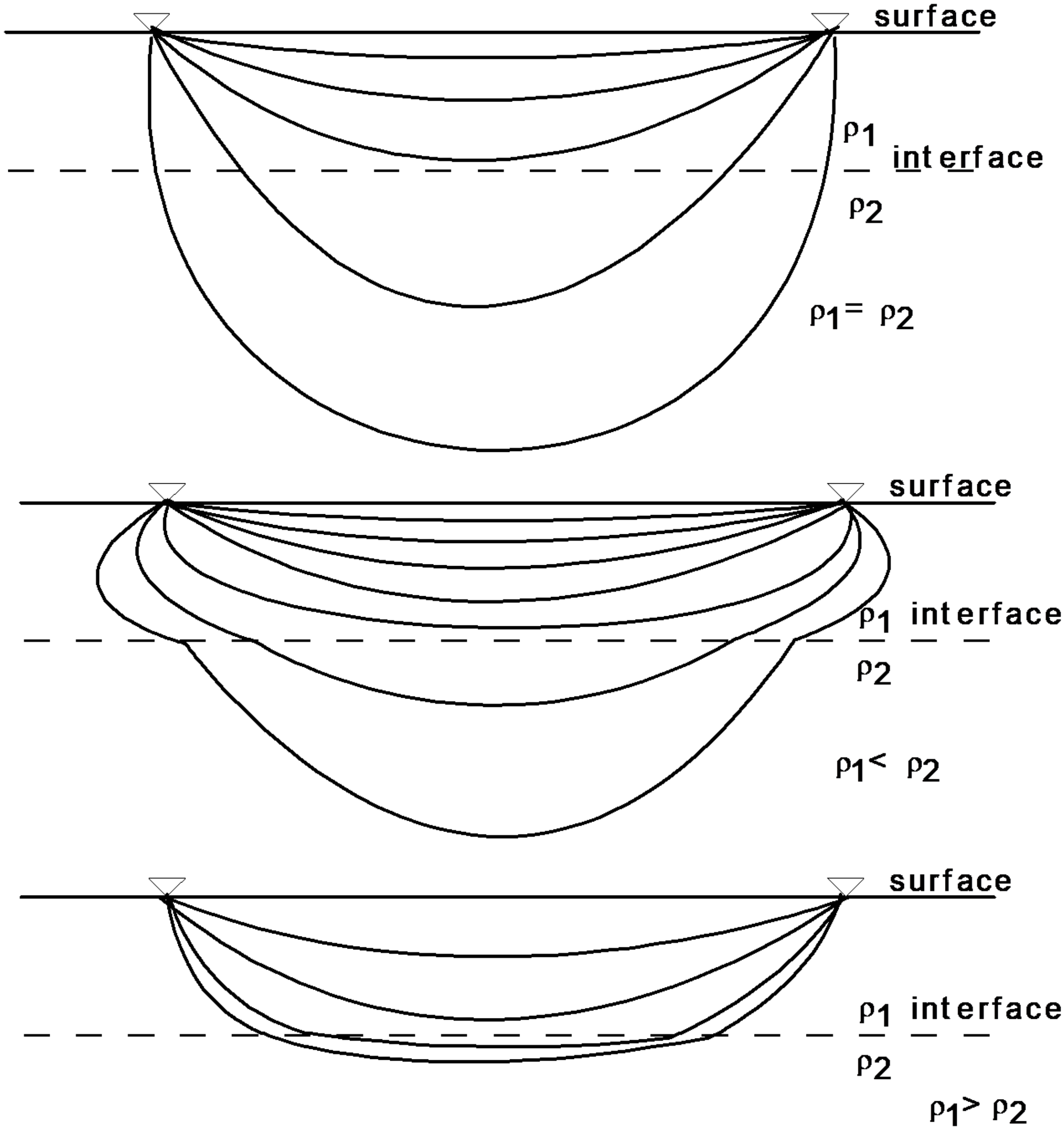
Since j=i/A, and i=ΔV/R, and ρ=RA/l, j=ΔV/RA, or j=ΔV/ρl. Thus, current density is proportional to potential within a tube extending along the flow line from the current electrode to the to the potential electrode. Does this mean that if we measure high voltages we can expect high currents? ! ? NO.

Consider measuring the potential between a wire and some point on the outside of insulation around that wire. We would measure a high potential difference between a point on the wire and a point outside the insulation - does that mean that the current across the wire through the insulation will be higher than the current through the wire?? !! What's wrong here?



The POTENTIAL difference is a function of the BATTERY - not the material. So the higher the voltage of the battery, the higher the current density, but, across an insulator, the current will be very low because the resistivity is very high.

Variations in current density near the earth's surface will be reflected in changes in potential difference, and will result in changes in apparent resistivity. Consider the cases below -

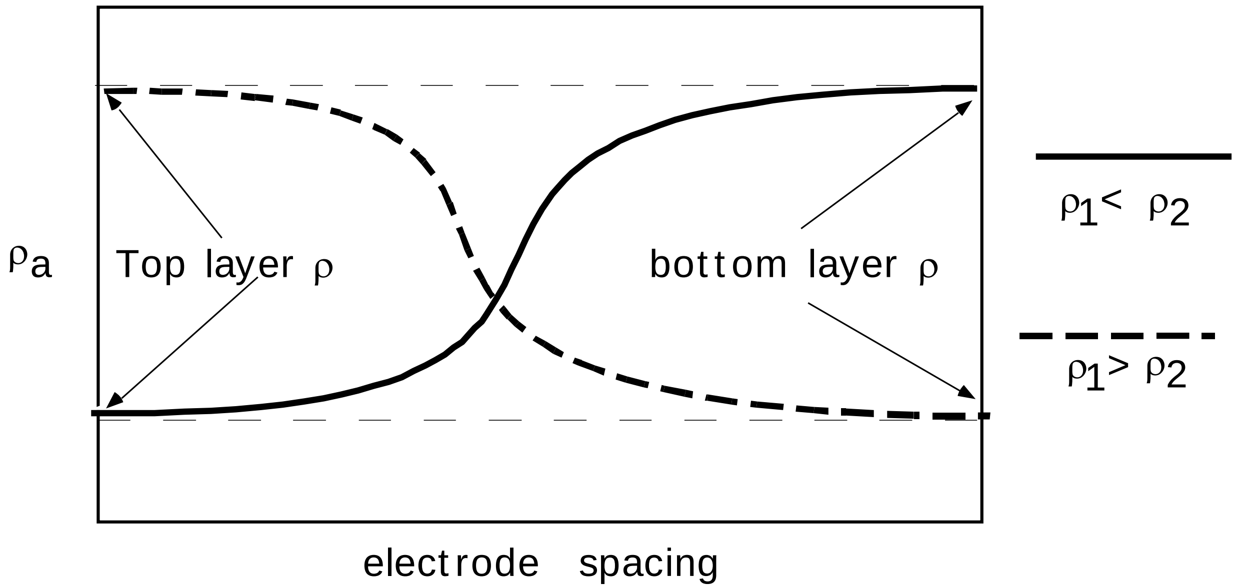


What happens if we move the layer up and down? If the interface is very deep, **RELATIVE TO THE ELECTRODE SPACINGS**, the lower layer should have no effect, and our readings shouldn't reflect its presence.

How deep is very deep?

What if we plot apparent resistivity vs. electrode spacing?

As spacing increases, we should "feel" deeper and deeper. At some point, if our electrodes are far enough apart, the top layer will have considerably less effect than the bottom



This change in apparent resistivity with electrode spacing should give us the information we need to interpret data and determine the depth to an interface and the resistivity of the materials.

ELECTRODE CONFIGURATIONS

The value of the apparent resistivity depends on the geometry of the electrode array used (K factor)

1- Wenner Arrangement
Named after wenner (1916) .
The four electrodes A , M , N , B are equally spaced along a straight line. The distance between adjacent electrode is called “a” spacing . So AM=MN=NB= ⅓ AB = a.

$$\mathbf{P_{a=}\ 2\,\pi\ a\quad V\ /\ I}$$

The wenner array is widely used in the western Hemisphere. This array is sensitive to horizontal variations.

2- **Lee- Partitioning Array** .
This array is the same as the wenner array, except that an additional potential electrode O is placed at the center of the array between the Potential electrodes M and N. Measurements of the potential difference are made between O and M and between O and N .

$$\mathbf{P_{a=}\ 4\,\pi\ a\quad V\ /\ I}$$

This array has been used extensively in the past .

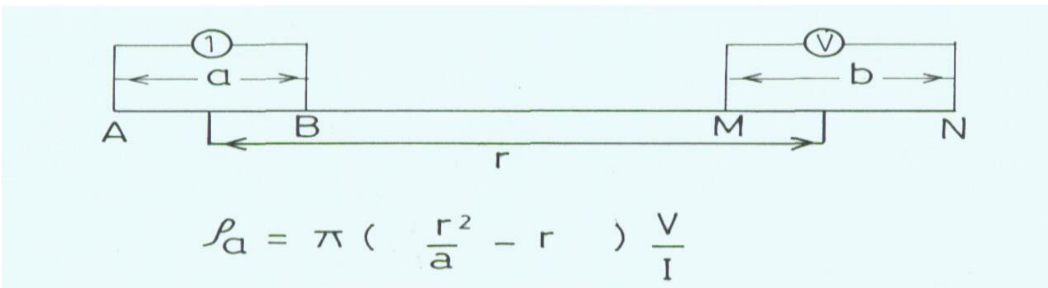
3) **Schlumberger Arrangement** .
This array is the most widely used in the electrical prospecting . Four electrodes are placed along a straight line in the same order AMNB , but with AB ≥ 5 MN

$$\rho a = \pi \times \frac{V}{I} \times \left[\frac{\left(\frac{AB}{2}\right)^2 - \left(\frac{MN}{2}\right)^2}{MN} \right]$$

This array is less sensitive to lateral variations and faster to use as only the current electrodes are moved.

1. Dipole – Dipole Array .

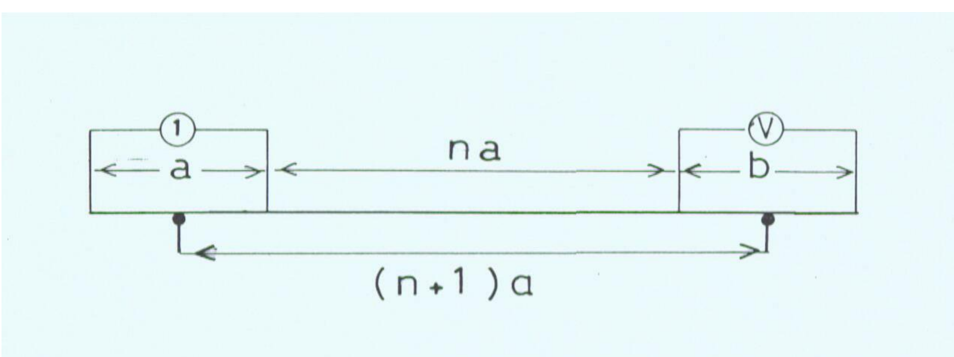
The use of the dipole-dipole arrays has become common since the 1950's , Particularly in Russia. In a dipole-dipole, the distance between the current electrode A and B (current dipole) and the distance between the potential electrodes M and N (measuring dipole) are significantly smaller than the distance r , between the centers of the two dipoles.



$$\boldsymbol{\rho_a = \pi \left[\left(r^2 / a \right) - r \right] v/i}$$

Or . if the separations a and b are equal and the distance between the centers is **(n+1) a** then

$$\boldsymbol{\rho_a = n\,(n+1)\,(n+2)\,.\,\pi\ a.\,v/i}$$



This array is used for deep penetration $\approx$ 1 km.

Four basic dipole- dipole arrays .

1) **Azimuthal**
2) **Radial**
3) **Parallel**
4) **Perpendicular**

When the azimuth angle **(Θ)** formed by the line r and the current dipole **AB = π /2** , The Azimuthal array and parallel array reduce to the equatorial Array.

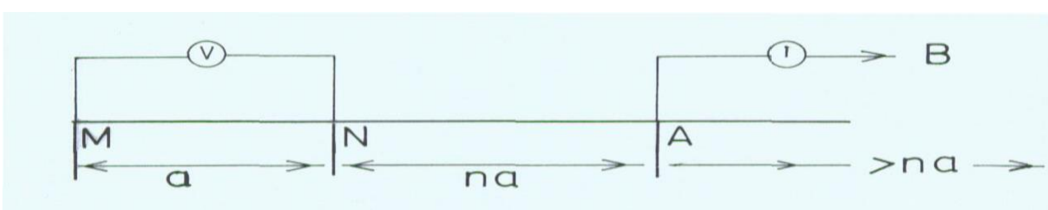
When **Θ = O** , the parallel and radial arrays reduce to the polar or axial array .

If MN only is small is small with respect to R in the equatorial array, the system is called Bipole-Dipole (**AB** is the bipole and **MN** is the dipole), where **AB** is large and MN is small.

If AB and MN are both small with respect to R , the system is dipole- dipole

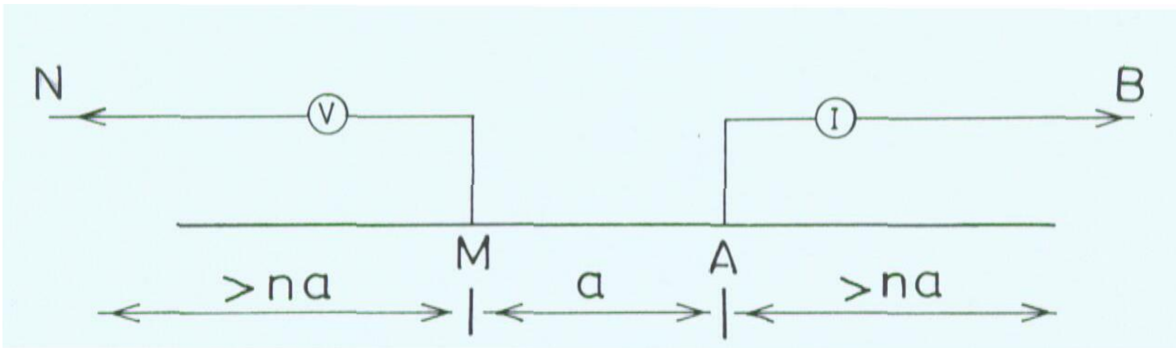
5) **Pole-Dipole Array .**
   The second current electrode is assumed to be a great distance from the measurement location (infinite electrode)

$$\rho_a = 2 \pi a n (n+1) \; v/i$$

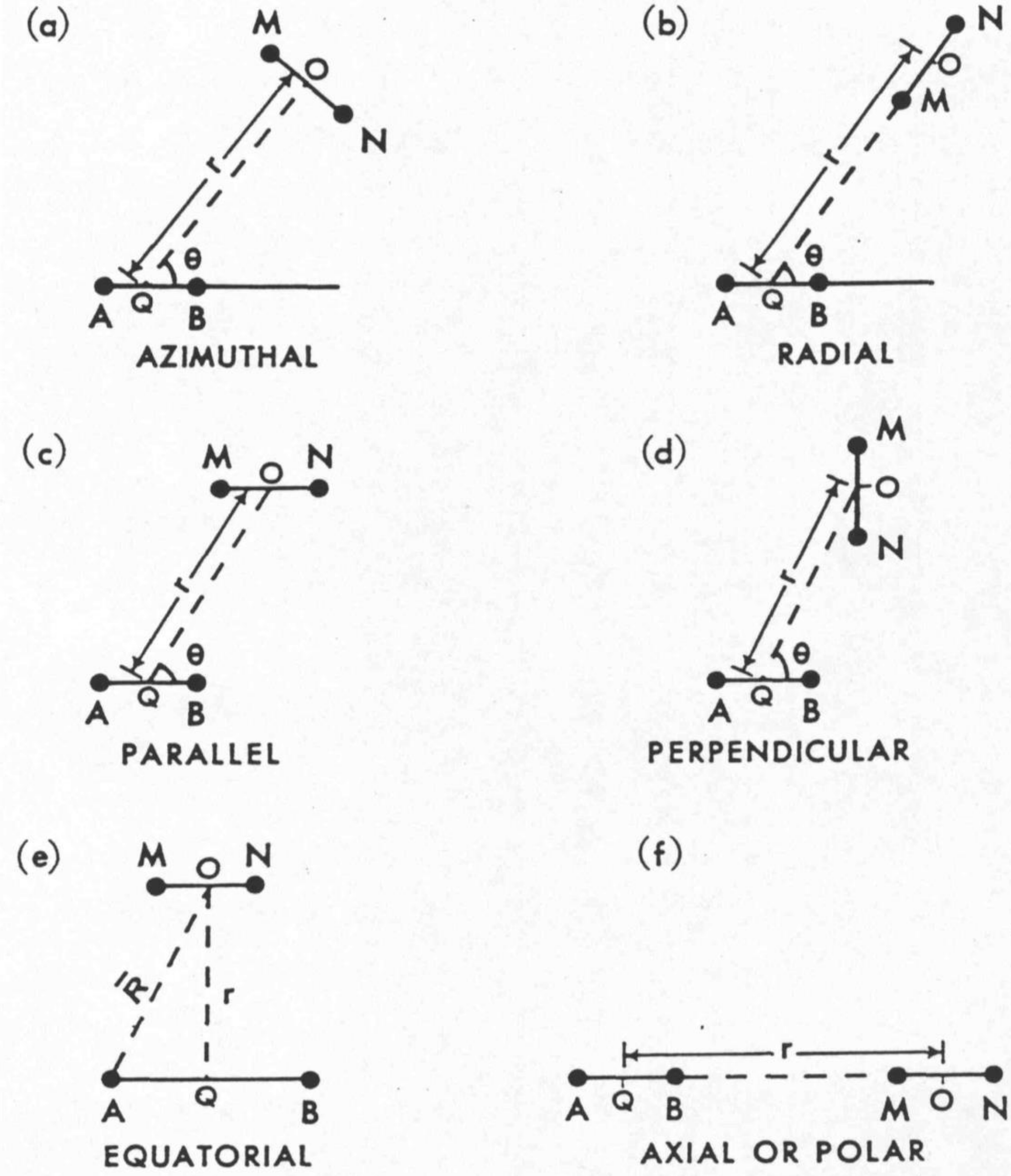


6) Pole – Pole.

If one of the potential electrodes , N is also at a great distance.



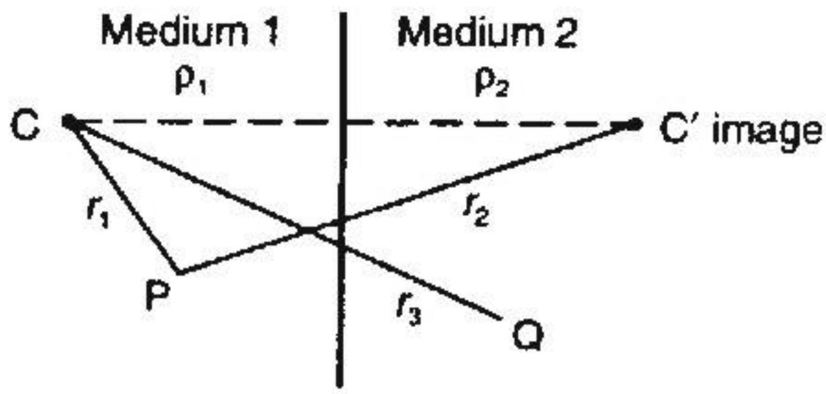
$P_{a=}$ 2π a V / I



Dipole-dipole arrays. The equatorial is a bipole-dipole array because AB is large.

Electrical Reflection Coefficient

Consider point current source and find expression for current potentials in medium 1 and medium 2: Use potential from point source, but 4π as shell is spherical:



Potential at point P in medium 1:

$$V_P = \frac{I\rho_1}{4\pi}\left[\frac{1}{r_1} + \frac{k}{r_2}\right]$$

Potential at point Q in medium2:

$$V_Q = \frac{I\rho_2}{4\pi}\frac{[1-k]}{r_3}$$

At point on boundary mid-way between source and its image:

$r_1 = r_2 = r_3 = r$ say. Setting $V_p = V_q$, and canceling we get:

$$\frac{\rho_1}{\rho_2} = \frac{1-k}{1+k} \qquad k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

k is electrical reflection coefficient and used in interpretation

The value of the dimming factor , K always lies between ± 1

If the second layer is a pure insulator
($\rho_2 = \omega$) then $K = + 1$

If the second layer is a perfect conductor

($\rho_2 = O$) then $K = - 1$

When $\rho_1 = \rho_2$ then No electrical boundary Exists and $K = O$.

SURVEY DESIGN

Two categories of field techniques exist for conventional resistivity analysis of the subsurface. These techniques are vertical electric sounding (VES), and Horizontal Electrical Profiling (HEP).

1- Vertical Electrical Sounding (VES) .

The object of VES is to deduce the variation of resistivity with depth below a given point on the ground surface and to correlate it with the available geological information in order to infer the depths and resistivities of the layers present.

In VES, with wenner configuration, the array spacing “a” is increased by steps, keeping the midpoint fixed (a = 2 , 6, 18, 54…….) .

In VES, with schlumberger, The potential electrodes are moved only occasionally, and current electrode are systematically moved outwards in steps

AB $\geq$ 5 MN.

2- Horizontal Electrical profiling (HEP) .

The object of HEP is to detect lateral variations in the resistivity of the ground, such as lithological changes, near- surface faults…… .

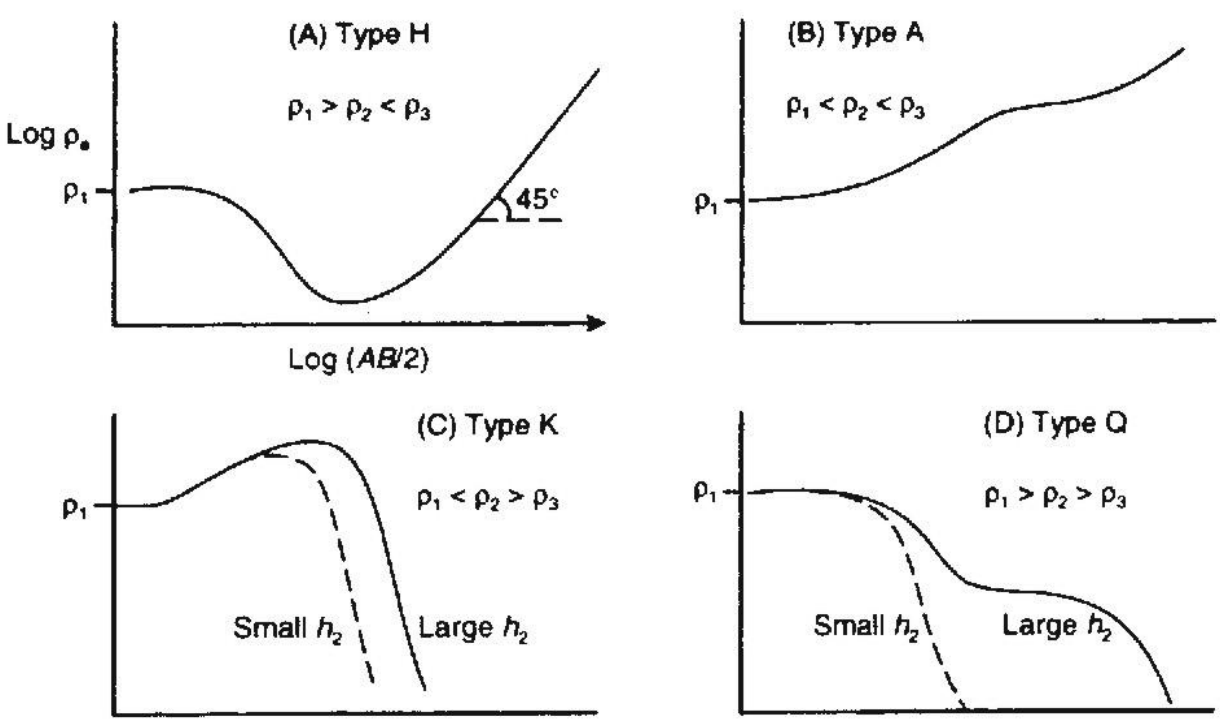
In the wenner procedurec of HEP , the four electrodes with a definite array spacing “a” is moved as a whole in suitable steps, say 10-20 m. four electrodes are moving after each measurement.

<u>In the schlumberger method of HEP,</u> the current electrodes remain fixed at a relatively large distance, for instance, a few hundred meters , and the potential electrode with a small constant separation (MN) are moved between A and B .

<u>Multiple Horizontal Interfaces</u>

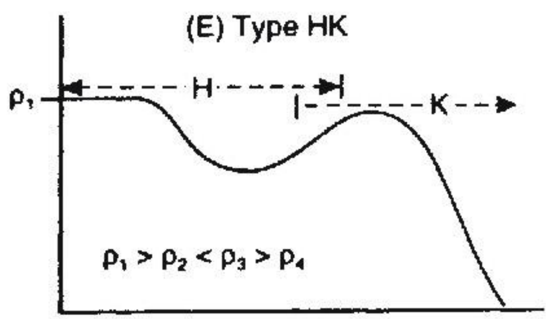
For Three layers resistivities in two interface case , four possible curve types exist.

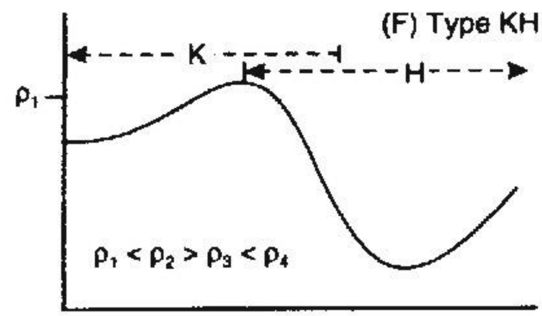
1- **Q – type** $\rho_1 > \rho_2 > \rho_3$
2- **H – Type** $\rho_1 > \rho_2 < \rho_3$
3- **K – Type** $\rho_1 < \rho_2 > \rho_3$
4- **A – Type** $\rho_1 < \rho_2 < \rho_3$



In four- Layer geoelectric sections, There are 8 possible relations :

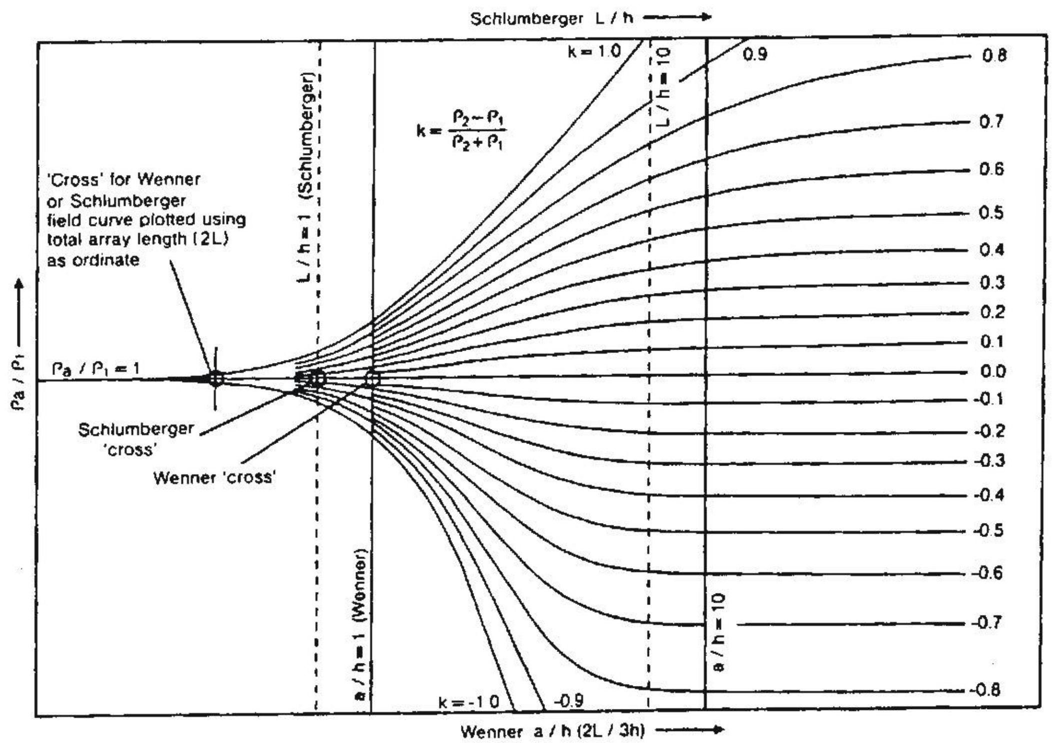
$\rho_1 > \rho_2 < \rho_3 < \rho_4$	**HA**	**Type**
$\rho_1 > \rho_2 < \rho_3 > \rho_4$	**HK**	**Type**
$\rho_1 < \rho_2 < \rho_3 < \rho_4$	**AA**	**Type**
$\rho_1 < \rho_2 < \rho_3 > \rho_4$	**AK**	**Type**
$\rho_1 < \rho_2 > \rho_3 < \rho_4$	**KH**	**Type**
$\rho_1 < \rho_2 > \rho_3 > \rho_4$	**KQ**	**Type**
$\rho_1 > \rho_2 > \rho_3 < \rho_4$	**QH**	**Type**
$\rho_1 > \rho_2 > \rho_3 > \rho_4$	**QQ**	**Type**





Quantitative VES Interpretation: Master Curves

Layer resistivity values can be estimated by matching to a set of master curves calculated assuming a layered Earth, in which layer thickness increases with depth. (seems to work well). For two layers, master curves can be represented on a single plot.



Master curves: log-log plot with ρ_a / ρ_1 on vertical axis and a / h on horizontal (h is depth to interface)

- Plot smoothed field data on log-log graph transparency.
- Overlay transparency on master curves keeping axes parallel.
- Note electrode spacing on transparency at which (a / h=1) to get interface depth.
- Note electrode spacing on transparency at which ($\rho_a / \rho_1 = 1$) to get resistivity of layer 1.
- Read off value of k to calculate resistivity of layer 2 from:

Quantitative VES Interpretation: Inversion

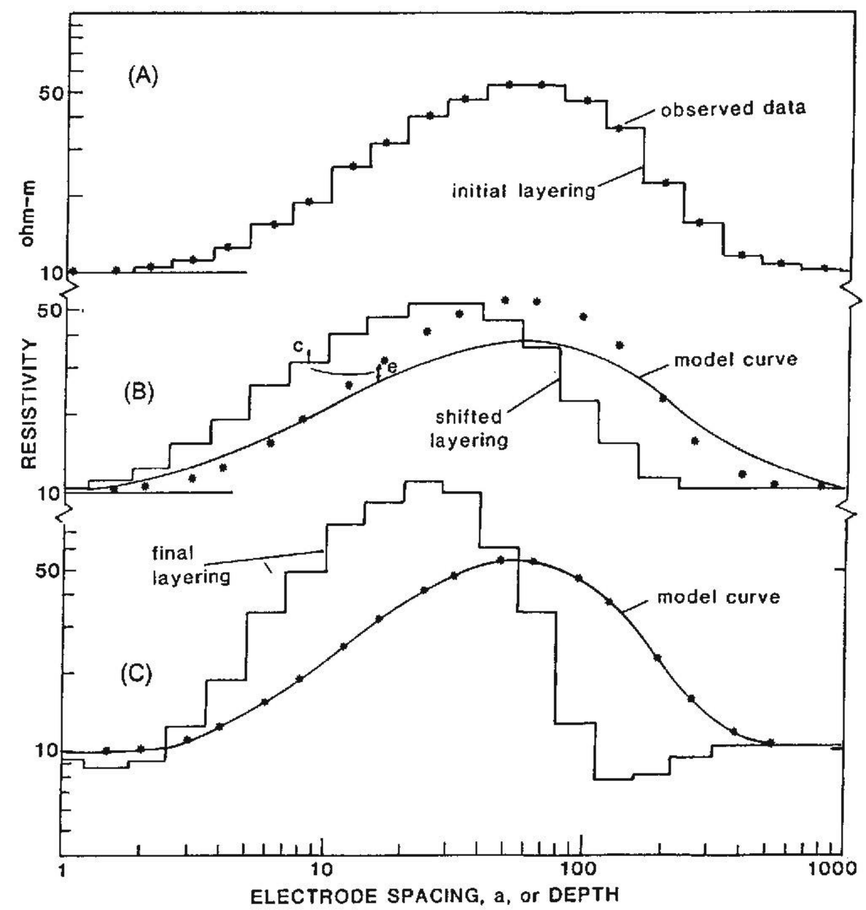
Curve matching is also used for three layer models, but book of many more curves.

Recently, computer-based methods have become common:

- forward modeling with layer thicknesses and resistivities provided by user
- inversion methods where model parameters iteratively estimated from data subject to user supplied constraints

Example (Barker, 1992)

Start with model of as many layers as data points and resistivity equal to measured apparent resistivity value.



Calculated curve does not match data, but can be perturbed to improve fit.

Applications of Resistivity Techniques

1. Bedrock Depth Determination

Both VES and CST are useful in determining bedrock depth. Bedrock usually more resistive than overburden. HEP profiling with Wenner array at 10 m spacing and 10 m station interval used to map bedrock highs.

2. Location of Permafrost

Permafrost represents significant difficulty to construction projects due to excavation problems and thawing after construction.

- Ice has high resistivity of 1-120 ohm-m

3. Landfill Mapping

Resistivity increasingly used to investigate landfills:

- Leachates often conductive due to dissolved salts
- Landfills can be resistive or conductive, depends on contents

Limitations of Resistivity Interpretation

1- **Principle of Equivalence**.

If we consider three-lager curves of K ($\rho_1 < \rho_2 > \rho_3$) or Q type ($\rho_1 > \rho_2 > \rho_3$) we find the possible range of values for the product $T_2 = \rho_2\ h_2$ Turns out to be much smaller. This is called T-equivalence. H = thickness, T : Transverse resistance it implies that we can determine T_2 more reliably than ρ_2 and h_2 separately. If we can estimate either ρ_2 or h_2 independently we can narrow the ambiguity. Equivalence: several models produce the same results. Ambiguity in physics of 1D interpretation such that different layered models basically yield the same response.

Different Scenarios: Conductive layers between two resistors, where lateral conductance (σh) is the same. Resistive layer between two conductors with same transverse resistance (ρh).

2- Principle of Suppression.

This states that a thin layer may sometimes not be detectable on the field graph within the errors of field measurements. The thin layer will then be averaged into on overlying or underlying layer in the interpretation. Thin layers of small resistivity contrast with respect to background will be missed. Thin layers of greater resistivity contrast will be detectable, but equivalence limits resolution of boundary depths, etc.

The detectibility of a layer of given resistivity depends on its relative thickness which is defined as the ratio of Thickness/Depth.

Comparison of Wenner and Schlumberger

(1) In Sch. MN ≤ 1/5 AB
Wenner MN = 1/3 AB

(2) In Sch. Sounding, MN are moved only occasionally.
In Wenner Soundings, MN and AB are moved after each measurement.

(3) The manpower and time required for making Schlumberger soundings are less than that required for Wenner soundings.

(4) Stray currents that are measured with long spreads effect measurements with Wenner more easily than Sch.

(5) The effect of lateral variations in resistivity are recognized and corrected more easily on Schlumberger than Wenner.

(6) Sch. Sounding is discontinuous resulting from enlarging MN.

Disadvantages of Wenner Array

1. All electrodes must be moved for each reading
2. Required more field time
3. More sensitive to local and near surface lateral variations
4. Interpretations are limited to simple, horizontally layered structures

Advantages of Schlumberger Array

1. Less sensitive to lateral variations in resistivity
2. Slightly faster in field operation
3. Small corrections to the field data

Disadvantages of Schlumberger Array

1. Interpretations are limited to simple, horizontally layered structures
2. For large current electrodes spacing, very sensitive voltmeters are required.

Advantages of Resistivity Methods

1. Flexible
2. Relatively rapid. Field time increases with depth
3. Minimal field expenses other than personnel
4. Equipment is light and portable
5. Qualitative interpretation is straightforward
6. Respond to different material properties than do seismic and other methods, specifically to the water content and water salinity

Disadvantages of Resistivity Methods

1- Interpretations are ambiguous, consequently, independent geophysical and geological controls are necessary to discriminate between valid alternative interpretation of the resistivity data (Principles of Suppression & Equivalence)

2- Interpretation is limited to simple structural configurations.

3- Topography and the effects of near surface resistivity variations can mask the effects of deeper variations.

4- The depth of penetration of the method is limited by <u>the maximum electrical power</u> that can be introduced into the ground and by the practical difficulties of laying <u>out long length</u> of cable. The practical depth limit of most surveys is about 1 Km.

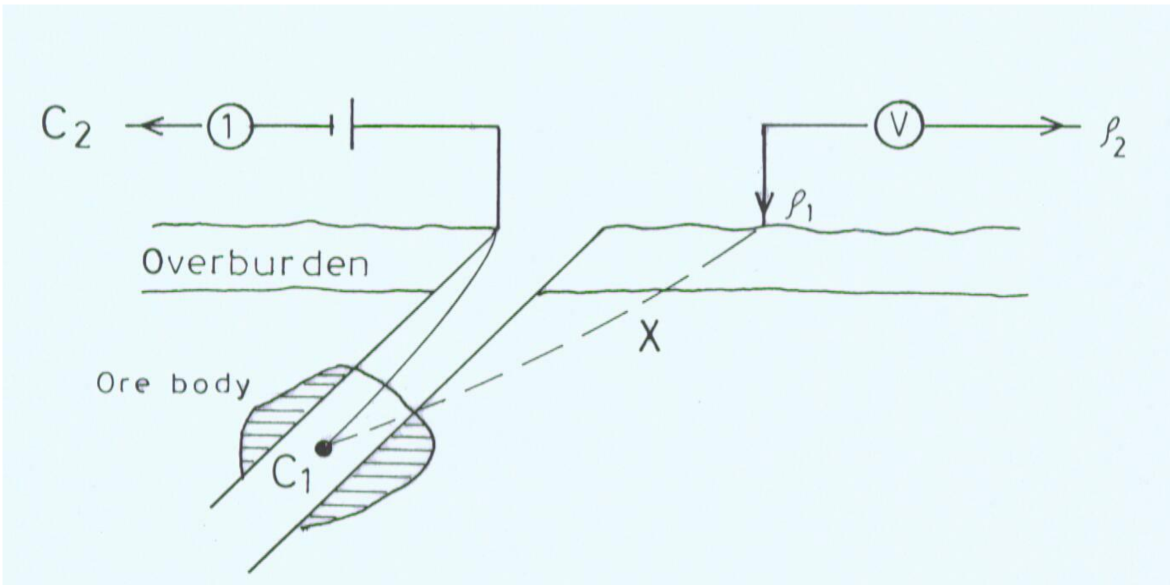
5. Accuracy of depth determination is substantially lower than with seismic methods or with drilling.

<u>Lateral inhomogeneities in the ground affect resistivity measurements in different ways: The effect depends on</u>

The size of inhomogeneity with respect to its depth
2. The size of inhomogeneities with respect to the size of electrode array
3. The resistivity contrast between the inhomogeneity and the surrounding media
4. The type of electrode array used
5. The geometric form of the inhomogeneity
6. The orientation of the electrode array with respect to the strike of the inhomogeneity

Mise-A-LA-Masse Method

This is a charged-body potential method is a development of HEP technique but involves placing one current electrode within a conducting body and the other current electrode at a semi- infinite distance away on the surface .



This method is useful in checking whether a particular conductive mineral-show forms an isolated mass or is part of a larger electrically connected ore body.

There are two approaches in interpretation

1- One uses the potential only and uses the maximum values a being indicative of the conductive body.

2- The other converts the potential data to apparent resistivity and thus a high surface voltage manifests itself in a high apparent resistivity

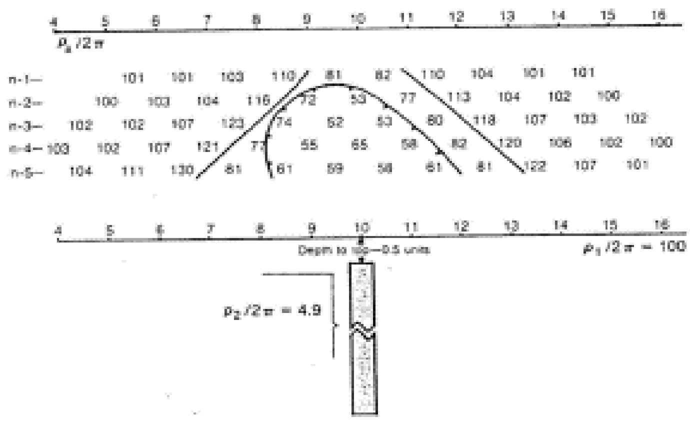
ρ_a = 4ЛI X V/I :

Where X is the distance between C1 and P1.

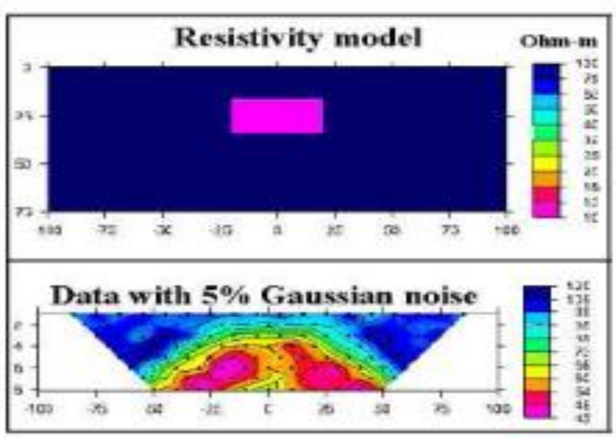
Pseudo-sections

A pseudo-section is *not* a true resistivity section – it is only a way of plotting the data.

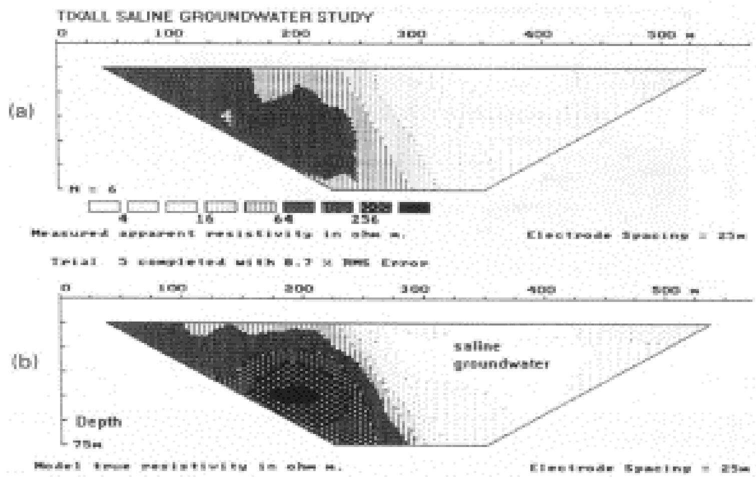
The data from a scale model illustrate the limitations – note the typical "inverted-Vee" shape of the result



Model study of pseudo-section



Pseudo-sections

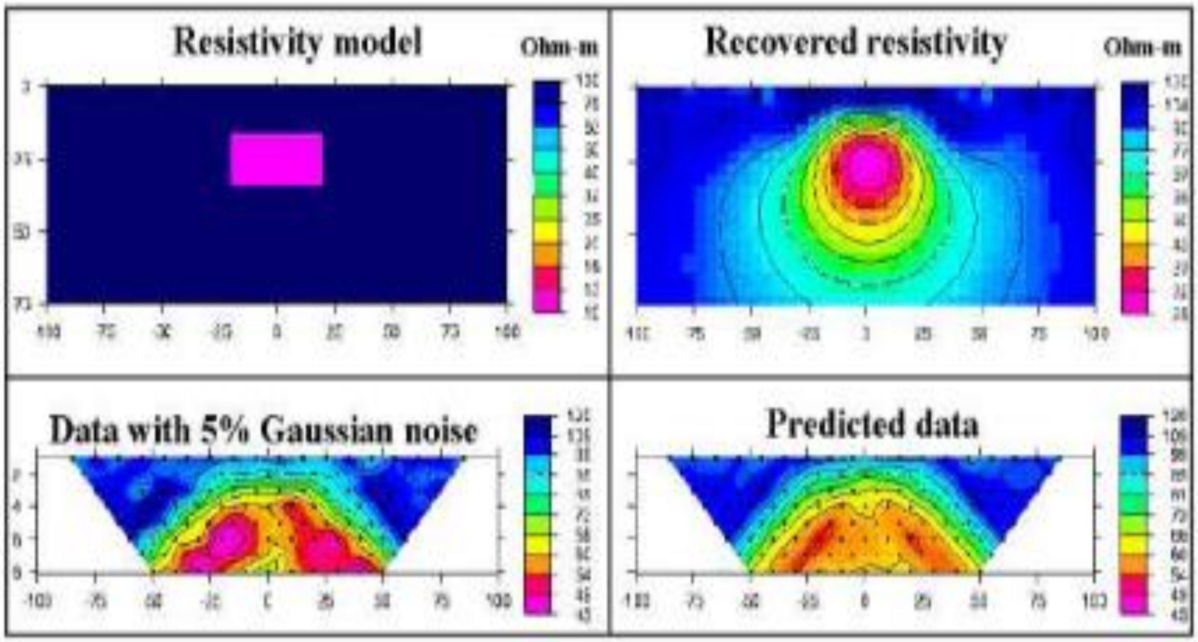


Top: a pseudo-section from a groundwater application.
Bottom: a "real-section" result using the same data.

"Pseudo-sections" vs "Real Sections"

- Pseudo-sections: an inaccurate image of the sub-surface
  - nothing more than a representation of the field data
- There are computer modelling methods for generating synthetic data, for a given 2D model of the resisitivity
- There are automatic methods for updating the model to be consistent with the data
  - generally known as "inverse modelling" methods
  - in resistivity/IP methods, the results are often referred to as "real sections"

Model study of “inversion”



Psuedo section

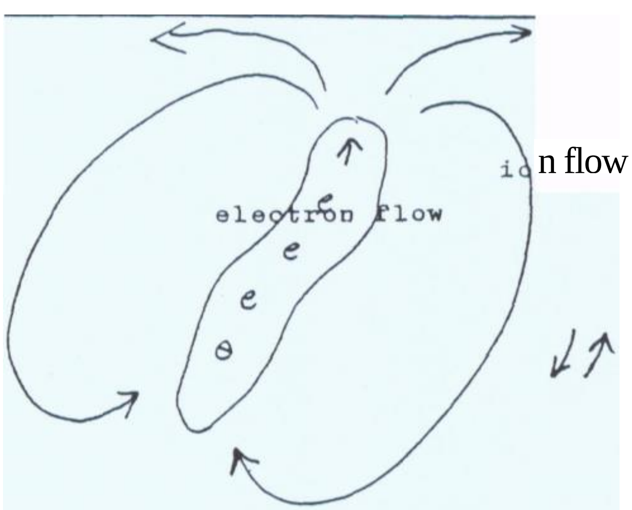
Inversion result, or “real” section

SELF- POTENTIAL (SP)

SP is called also spontaneous polarization and is a naturally occurring potential difference between points in the ground. SP depends on small potentials or voltages being naturally produced by some massive ores.
The self-potential method makes use of natural currents flowing in the ground that are generated by electrochemical processes to locate shallow bodies of anomalous conductivity and water circulation.
It associate with sulphide and some other types of ores. It works strongly on pyrite, pyrrohotite, chalcopyrite, graphite.
SP is the cheapest of geophysical methods.

Conditions for SP anomalies

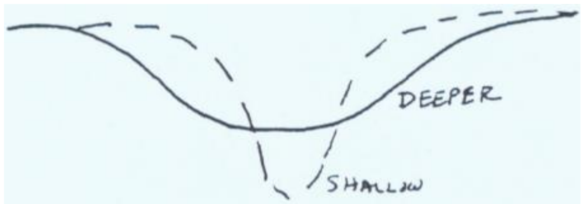
1) Shallow ore body
2) Continuous extension from a zone of oxidizing conditions to one of reducing conditions, such as above and below water table.

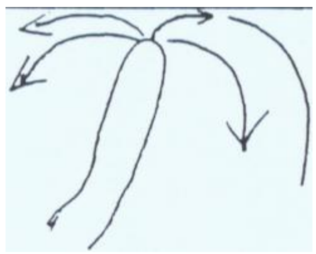


either or both
pos/neg ions

Note that it is not necessary that an individual ion travel the entire path. Charges can be exchanged.

The implications of this for potential distribution would be

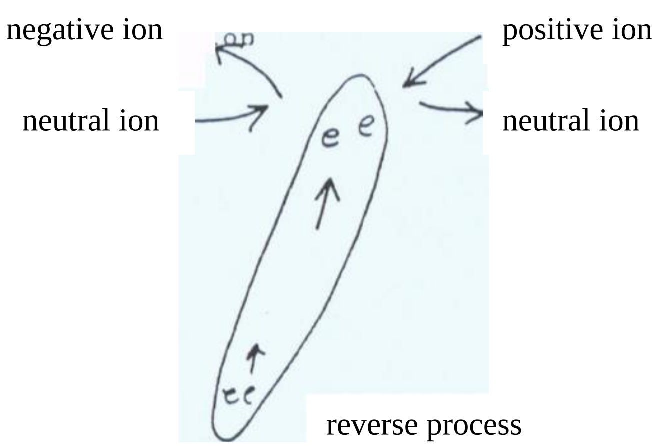




When we come to consider more specifically the mechanism, we see that it must be consistent with

- ➔ - electron flow in the ore body
- ➔ - ion flow in surrounding rock
- ➔ - no transfer of ions across ore boundary, although electrons are free to cross

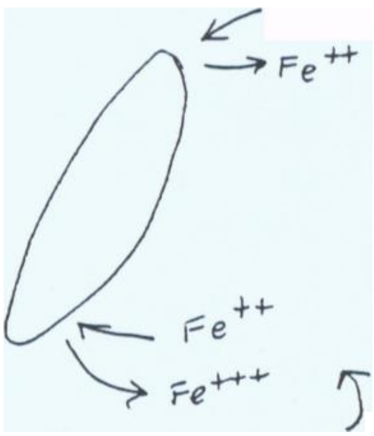
That is we must have



When we consider the possible ion species, the criteria would be

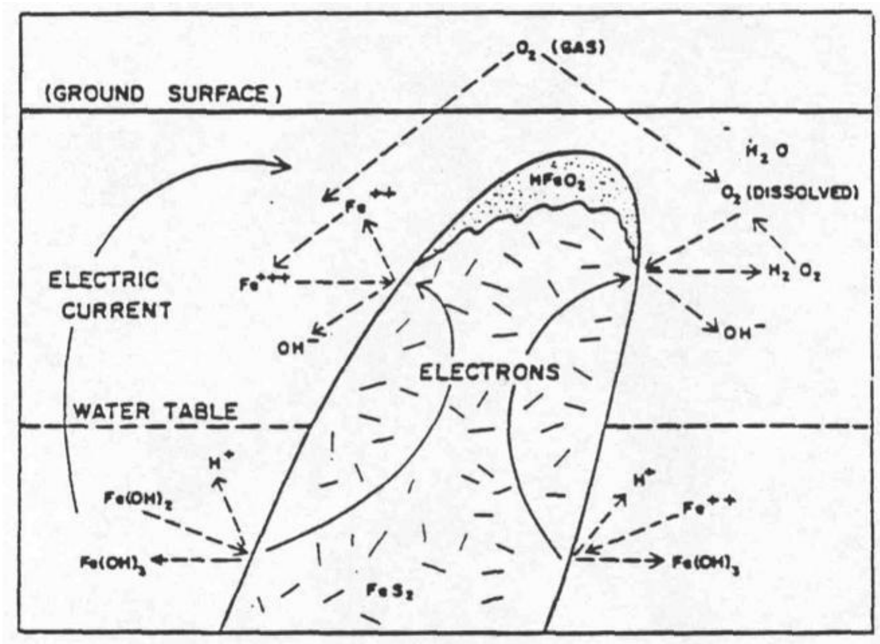
- common enough
- reversible couple under normal ground conditions
- mobile enough

Sato and Mooney proposed ferric/ferrous couples to satisfy these criteria.



made continuous by $O_2 - H_2O_2$ reaction with O_2 supplied from atmosphere

made continuous by reactions involving ferrous and ferric hydroxide with presence of H^+



Proposed electrochemical mechanism for self-potentials

This proposed mechanism have two geologic implications:

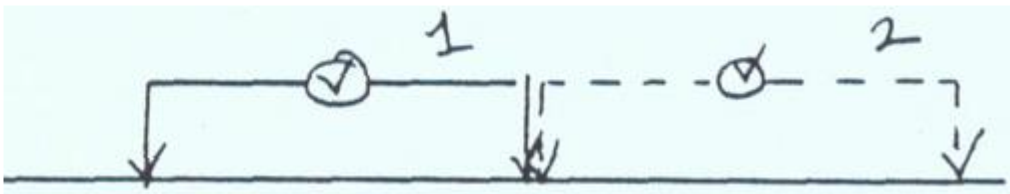
1) The ore body must be an electronic conductor with high conductivity. This would seem to eliminate sphalerite (zinc sulfide) which has low conductivity.

2) The ore body must be electrically continuous between a region of oxidizing conditions and a region of reducing conditions. While water table contact would not be the only possibility have, it would seem to be a favorable one.

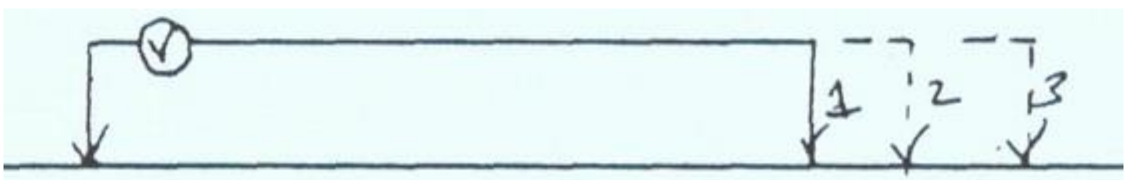
Instrumentation and Field Procedure

Since we wish to detect currents, a natural approach is to measure current. However, the process of measurement alters the current. Therefore, we arrive at it though measuring potentials.

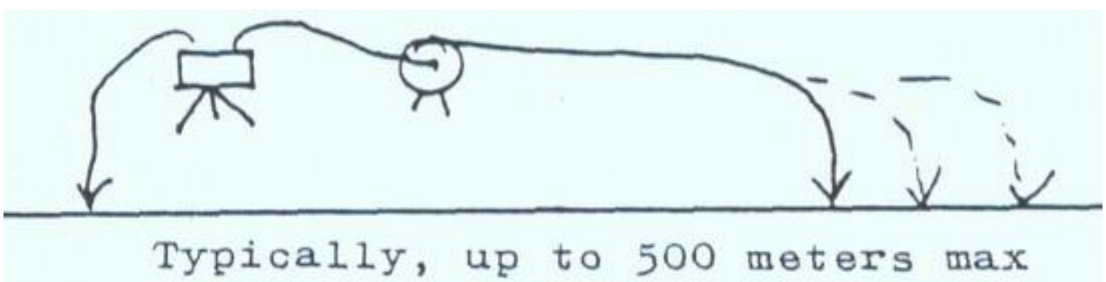
Principle, and occasional practice:



More usual practice



Instruments



Equipment:

- potentiometer or high impedance voltmeter
- 2 non-polarizing electrodes
- wire and reel

Non-polarizing electrodes were described in connection with resistivity exploration although they are not usually required there. Here, they are essential. The use of simple metal electrodes would generate huge contact or corrosion potentials which would mask the desired effect. non-polarizing electrodes consist of a metal in contact with a saturated solution of a salt of_the metal . Contact with the earth can be made through a porous ceramic pot.

The instrument which measures potential difference between the electrodes must have the following characteristics:

a) capable of measuring +0.1 millivolt,
b) capable of measuring up to ±1000 millivolts (±1 volt)
c) input impedance greater than 10 megaohms, preferably more.

The high input impedance is required in order to avoid drawing current through the electrodes, whose resistance is usually less than 100 kilohms. In very dry conditions (dry rock, ice, snow, frozen soil), the electrode resistance may exceed 100 kilohms, in which case the instrument input impedance should also be increased.

SP are produced by a number of mechanisms :

1) **Mineral potential** (ores that conduct electronically) such as most sulphide ores ,Not sphalerite (zinc sulphide) magnetite, graphite. Potential anomaly over sulfide or graphite body is negative The ore body being a good conductor. Curries current from oxidizing electrolytes above water – table to reducing one below it .

2) Diffusion potential

$$E_d \quad \frac{RT(I_a - I_c)}{n\,F(I_a + I_c)} \qquad Ln\,(C_1 / C_2)$$

Where

I_a , I_c Mobilities of the anions (+ve) and cations(-ve)
R= universal Gas constant (8.314JK^{-1} mol^{-1})
T : absolute temperature (K)
N : is ionic valence
F: Farady's constant 96487 C mol^{-1})
C_1 , C_2 Solution concentrations .

3) Nernst Potential

$$E_N = -(RT / nF) \, Ln (C_1 / C_2)$$

Where $I_a = I_c$ in the diffusion potential Equation .

4. **Streaming potentials** due to subsurface water flow are the source of many SP anomalies. The potential E per unit of pressure drop P (The streaming potentials coupling cocfficent) is given by :

$$E_K = -\frac{\varepsilon \, \rho \, C_E \, \delta P}{4 \, \pi \, \eta}$$

ρ Electrical Resistivity of the pore Fluld.
E_k Electro-kinetic potential as a result from an electrolyte flowing through a porous media.
ε Dielectric constant of the pole fluid.
η Viscosity of the pore fluid
δP pressure difference
C_E electro filtration coupling coefficient.

If the grain size decreases, C increases

- If the temperature decreases, C decreases
- If viscosity decreases, C increases
- Permeability has a complex effect on C

.

Advantages :

Survey simple
Non expensive
Allows for a rapid qualitative mapping of the underground
Suitable for monitoring

Disadvantages

- Very sensitive to noise
- Physical aspects still not well understood
- Quantitative aspects still need to be develop

Interpretation

Usually, interpretation consists of looking for anomalies.

The order of magnitude of anomalies is

0-20 mv	normal variation
20-50 mv	possibly of interest, especially if observed over a fairly large area
over 50 mv	definite anomaly
400-1000 mv	very large anomalies

Depth of investigation depends on the size of the mineralized body and the depth of the water table for a mineralization potential (generally shallow, < 30 m)

- Interpretation mainly qualitative (profile, map)
- Quantitative using dipole approximations for the polarized

body (similar to magnetic interpretation)

Applications

Groundwater applications rely principally upon potential differences produced by pressure gradients in the groundwater. Applications have included
detection of leaks in dams and reservoirs
location of faults, voids, and rubble zones which affect groundwater flow
delineation of water flow patterns around landslides, wells, drainage structures, and springs, studies of regional groundwater flow

Other groundwater applications rely upon potential differences produced by gradients in chemical concentration, Applications have included
outline hazardous waste contaminant plumes

Thermal applications rely upon potential differences produced by temperature gradients.
Applications have included
geothermal prospecting
map burn zones for coal mine fires
monitor high-temperature areas of in-situ coal gasification processes and oil field steam and fire floods.

Induced Polarization (IP)

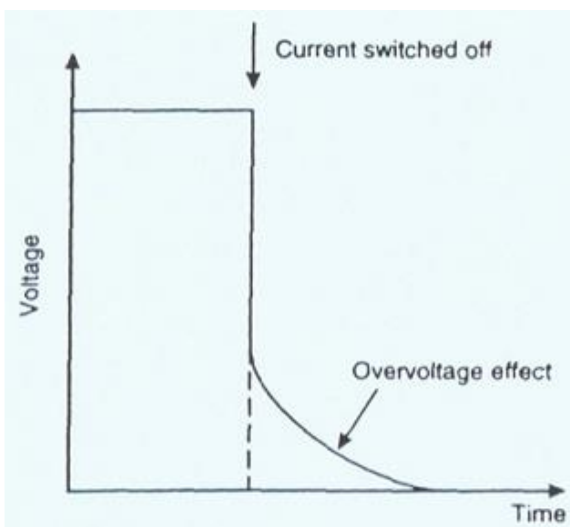
IP depends on a small amount of electric charge being stored in an ore when a current is passed Through it , to be released and measured when the current is switched off .

The main application is in the search for **disseminated metallic ores** and to a lesser extent, ground water and geothermal exploration .

Measurements of IP using 2 current electrodes and 2 non-polarizable potential electrodes. When the current is switched off , the voltage between the potential electrodes takes a finite to decay to zero because the ground temporarily stores charge **(become Polarized**)

Four systems of IP .

1- Time domain
2- Frequency domain < 10 HZ
3- Phase domain
4- Spectral IP 10^{-3} to 4000 HZ



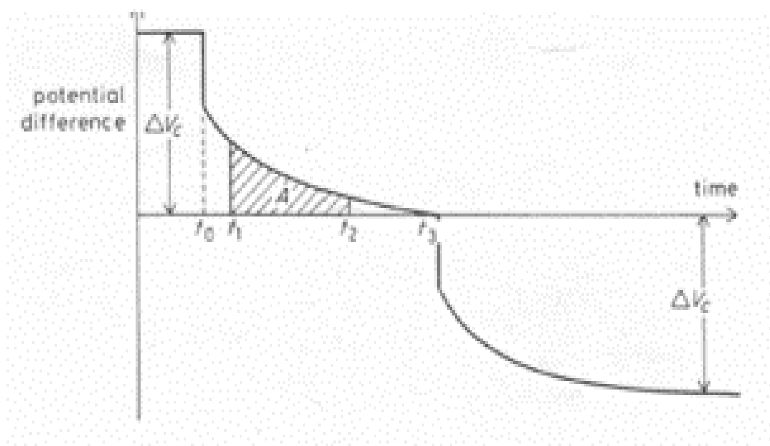
Factors affecting induced polarization	
Membrane polarization	Electrode polarization
Clay content	Metallic mineral concentration
Clay mineralogy	Type, size of grains
Porosity	Porosity
Fluid salinity	Fluid salinity
Temperature	Temperature

Induced polarization (IP method)

IP Effect:

Switch off current, voltage decays

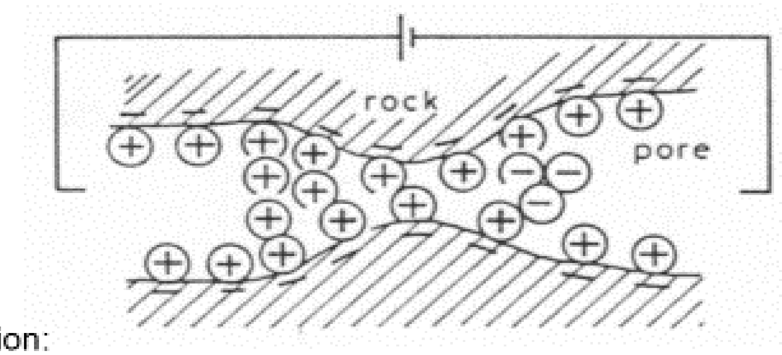
Switch on current, voltage builds up



This is "time-domain" IP – at t_0 current is switched off, the IP effect is measured by measuring the area A under the decay curve.

Alternative is "frequency-domain IP" – see later

Mechanisms causing the IP effect



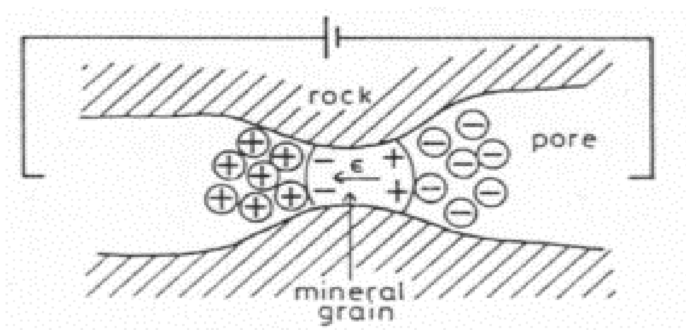
Membrane polarization:

• in clays, electrolytic current flow is impeded by positive ions (attracted to negatively charged clay particles)

• charges accumulate, voltage builds up

• on release of current, charges drift back to equilibrium, voltage decays

Mechanisms causing the IP effect



Electrode polarization:

• metallic mineral grains conduct electronically, electrolytic ions accumulate at pore restrictions causing a buildup of voltage

• on current release, ions drift to equilibrium positions, leading to a voltage decay

IP Effect – groundwater and environmental applications

- Membrane polarization observed in clay minerals – can be used as an indicator of clays
- Applications in aquifer studies, monitoring of clay-organic processes in organic contamination
- Salinity affects the strength of the effect
- Metal remnants, galvanic sludge, glazed ceramics are also chargeable

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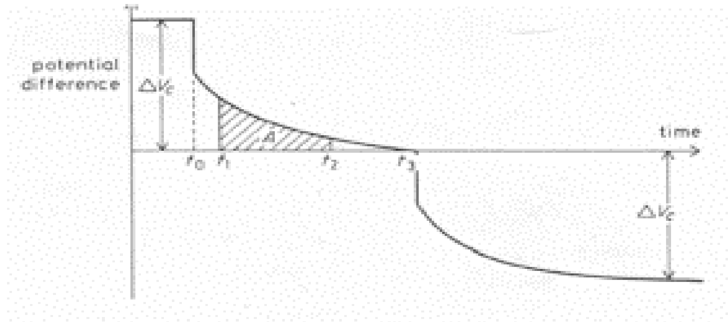
IP Effect – mineral exploration

- The greater the exposed metallic surface area, the stronger the effect
- Enhanced effect for disseminated mineral grains
- Often these are cases in which standard resistivity response is weak
- In mineral exploration, the effect of near-surface clays complicates the measurement

Time-domain IP measurement

$$M = \frac{1}{\Delta V_o} \int_{t_1}^{t_2} v(t)\, dt$$

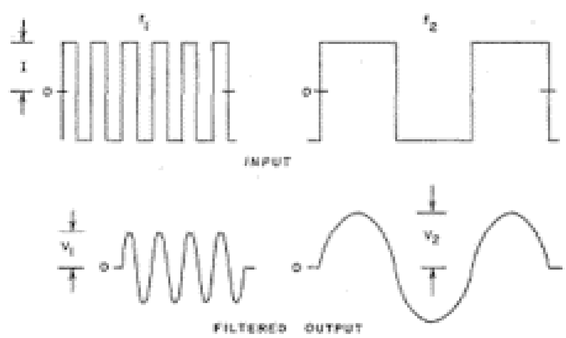


Use electronics to measure the area (i.e., the integral) A:

This is called the “chargeability” (measured in ms)

Typically t_1~0.5 s and t_2~1.0 s

Frequency domain measurement of Induced Polarization



- A square wave current source is switched at two frequencies
- The AC output voltage is measured for each input frequency
- At low frequency, the output voltage stabilizes to the DC response before the voltage is switched
- At high frequency the output voltage is lower, as the input switches before the voltage stabilizes

Frequency-domain IP measurement

- At very low switching frequencies, the voltage has time to build up to a level V_{DC}
- At high switching rates, the voltage does not build up to V_{DC}
- Instead we measure a lower voltage, V_{AC}
- For each we can define an apparent resitivity:

$$\rho_{DC} = 2\pi k \frac{V_{DC}}{I} \quad \text{and} \quad \rho_{AC} = 2\pi k \frac{V_{AC}}{I}$$

- The apparent resisitivity thus decreases with frequency in the presence of an IP effect

Frequency-domain IP measurement

- The apparent resisitivity thus decreases with frequency in the presence of an IP effect
- We define the “Frequency-effect” as

$$FE = \frac{\rho_{DC} - \rho_{AC}}{\rho_{AC}}$$

- In conductive areas the charge buildup is partially short-circuited, reducing the FE
- This is compensated for in the “Metal Factor”:

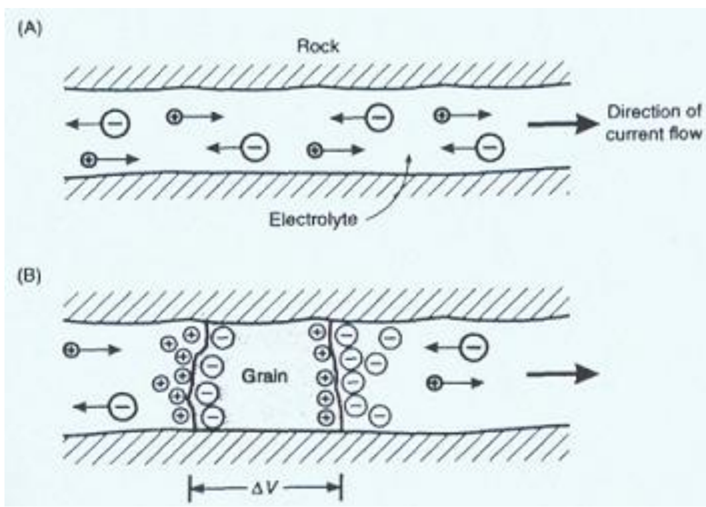
$$MF = \frac{10^3 \, PFE}{\rho_{DC}} \quad [\text{Siemens/m}]$$

Sources of IP Effects

1) **Normal IP**
   - Membrane Polarization
   - Most Pronounced with clays
   - Decreases with very high (> 10%) clay content due to few pores, low conductivity.

2) **Electrode polarization**
   - Most metallic minerals have EP
   - Decreases with increased porosity.
   - Over-voltage effect

3) **IP is A bulk effect.**



Grain (electrode) polarization. (A) Unrestricted electrolytic flow in an open channel. (B) Polarization of an electronically conductive grain, blocking a channel

1. Time – domain measurements.

One measure of the IP effects is the ratio V_p / V_o which is known chargeability which expressed in terms of millivolts per volt or percent.

V_p : overvoltage
V_o : observed voltage

$$\mathbf{M = V_p / V_o \ (m_v / v \text{ or } \%)}$$

Apparent chargeability

$$M_a = (1 / V_0) \int_{t1}^{t2} V_p(t)\, dt = A / V_0$$

V_p (t) is the over-voltage at time t .

10 – 20 %	sulphides	1000-3000 msec .
	Sand stones	100-200 msec.
	Shale	50-100
	Water	0

2) Frequency- Domain measurements.

Frequency effect **FE= (P_{ao} –P_{a1}) / P_{a1}** (unitless)

P_{ao} : apparent resistivity at low frequencies

P_{al} : appatent resistivity at high frequencies

$$\mathbf{P_{ao} > P_{a1}}$$

Percentage frequency affect **PFE = 100(P_{ao} –P_{a1}) / P_{a1} = 100 FE**

The frequency effect in the frequency domain is equivalent to the chargeability in the time domain for a weakly polarisable medium where FE < 1 .

Metal Factor MF= A (ρ_{a0} – ρ_{a1}) / (ρ_{a0} ρ_{a1})

= A (δ_{a1} – δ_{a0}) siemens / m

ρ_{a0} & ρ_{a1} apparent resistivity.

δ_{a0} and δ_{a1} are apparent conductivities ($1/\rho_a$) at low and higher frequencies respectively where

$\rho_{a0} > \rho_{a1}$ **and** $\delta_{a0} < \delta_{a1}$ $\qquad$ $A = 2\pi \times 10^5$

$$MF = A \times FE / \rho_{a0} = A \times FE / \rho_{a0}$$

$$= FE / \rho_{a0} = A \times FE \times \delta_{a0}$$

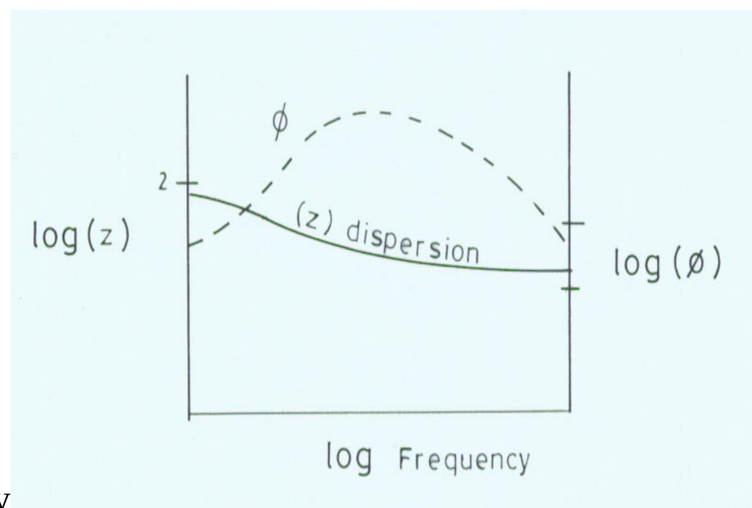
The above methods do not give a good indication of the relative amount of the metallic mineralization within the source of the IP. It is necessary to go with spectral IP.

3. Spectral IP and Complex Resistivity.

Is the measurement of the dielectric properties of materials

Ө is the phase lag between the applied current and the polarization voltage measured.

$$|z(w)| = P_0 [1 - M (1 - 1/ (1+(iw\tau)^c)]$$



Z(w) : complex resistivity

P_0 : D.c. resistivity

M : IP chargeability

W : Angular frequency.

τ : Time constant. (relaxation time) is the behaviour between the lower and upper frequency limits.

i : √ -1

c : frequency exponent

Critical Frequency (F_c) : Which is the specific frequency at which the maximum phase shift is measured. This frequency is completely independent of resistivity.

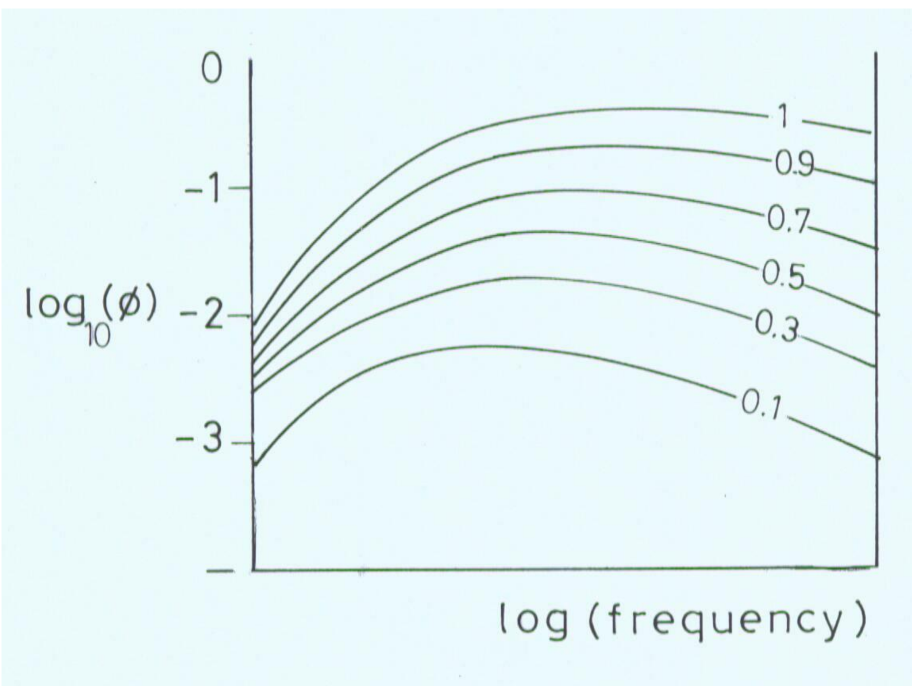
Phase angle and the critical frequency increase with increasing chargeability.

$$F_c = [2\pi \tau (1 - M)^{1/2c}]^{-1}$$

τ Time constant

M IP chargeability .

This is call cole – cole relalaxation



IP Survey Design

1- Profiling : Later contrasts in electrical properties such as lithologic contacts. (wenner + Dipole – Dipole) .

2- Sounding : to map the depths and thickness of stratigraphic units (Schlumberger + wenner).

3- Profiling – Sounding : in contaminant plume mapping , where subsurfae electrical propertios are expected to vary vertically and horizontally (wenner + Dipole – Dipole) .

<u>Limitations of IP</u>

1- IP is more susceptible to sources of cultural interference (metal fences, pipe lines , power lines) than electrical resistivity.

2- IP equipment requires more power than resistivity alone . This translates into heavier field instruments

3- The cost of IP much greater than resistivity – alone system.

4- IP requires experience.

5- Complexity in data interpretation.

6- Intensive field work requires more than 3 crew members.

7- IP requires a fairly large area far removed from power lines , fences, pipelines .

Advantages of IP

1- IP data can be collected during an electrical resistivity survey

2- IP data and resistivity together improves the resolution of the analysis of Resistivity data in three ways:

   a. some of the ambiguities in resistivity data can be redueed by IP analysis.

   b. IP can be used to distinguish geologic layers which do not respond well to an electrical resistivity .

   c. Measurements of chargeability can be used to discriminate equally electrically conductive target such as saline, electrolytic or metallic-ion contaminant plumes from clay Layers.

ELECTROMAGNETIC METHODS

Introduction

Electromagnetic methods in geophysics are distinguished by:

1. Use of differing frequencies as a means of probing the Earth (and other planets), more so than source-receiver separation. Think “skin depth”. Sometimes the techniques are carried out in the frequency domain, using the spectrum of natural frequencies or, with a controlled source, several fixed frequencies (FDEM method ---“frequency domain electromagnetic”). Sometimes the wonders of Fourier theory are involved and a single transient signal (such as a step function) containing, of course, many frequencies, is employed (TDEM method - “time domain electromagnetic”). The latter technique has become very popular.

2. Operate in a low frequency range, where conduction currents predominate over displacement currents. The opposite is true (i.e., has to be true for the method to work) in Ground Penetrating Radar (GPR). GPR is a wave propagation phenomenon most easily understood in terms of geometrical optics. Low frequency EM solves the diffusion equation.

3. Relies on both controlled sources (transmitter as part of instrumentation) and uncontrolled sources. Mostly the latter is supplied by nature, but also can be supplied by the Department of Defense.

- EM does not require direct Contact with the ground. So, the speed with EM can be made is much greater than electrical methods.

- EM can be used from aircraft and ships as well as down boreholes.

Advantages

- lightweight & easily portable.
- Measurement can be collected rapidly with a minimum number of field personnel .
- Accurate
- Good for groundwater pollution investigations.

Limitations :

- Cultural Noise

Applications

1. Mineral Exploration
2. Mineral Resource Evaluation
3. Ground water Surveys
4. Mapping Contaminant Plumes
5. Geothermal Resource investigation
6. Contaminated Land Mapping
7. Landfill surveys
8. Detection of Natural and Artificial Cavities
9. Location of geological faults
10. Geological Mapping

Type of EM Systems

- EM can be classified as either :
    1. Time – Domain (TEM) or
    2. Frequency- Domain (FEM)

- FEM use either one or more frequencies.
- TEM makes measurements as a function of time .
- EM can be either :
  a- Passive, utilizing natural ground signals (magnetotellurics)
  b- Active , where an artificial transmitter is used either in the near-field (As in ground conductivity meters) or in the far-field (using remote high-powered military transmitters as in the case of VLF Mapping 15-24 KHZ).

Factors Affecting EM Signal

The signal at the Receiver depends on :

1) the material 2) Shape 3) Depth of the Target

4) Design and positions of the transmitter and receiver coils .

The size of the current induced in the target by the transmitter depends on

1) Number of lines of magnetic field through the Loop (magnetic flux) 2) Rate of change of this number 3) The material of the loop.

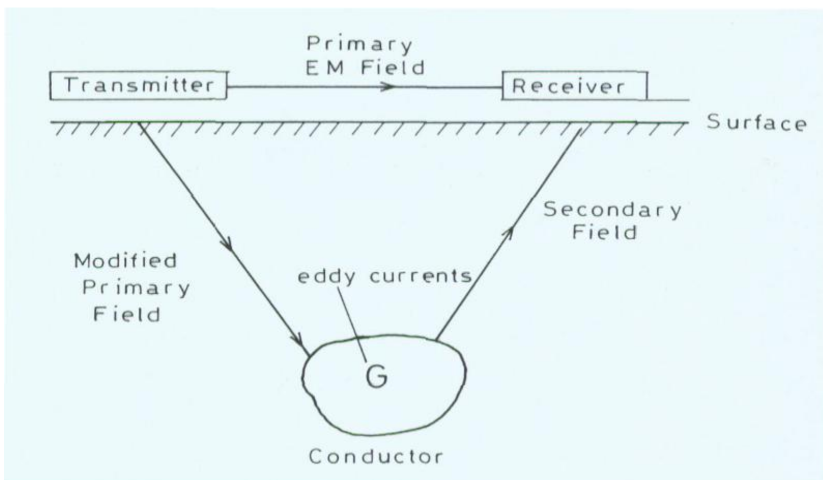
Magnetic flux Depends on :

1) The Strength of the magnetic field at the Loop 2) Area of the Loop 3) Angle of the loop to the field

Flux Ø = Magnetic field X cos Ө X area X number of turns .

Principle of EM surveying

- EM field can be generated by passing an alternating current through either a small coil comprising many turns of wire or a large loop of wire .
- The frequency range of EM radiation is very wide, from < 15 HZ (atmospheric micropulsations) , Through radar bands (10^8 – 10^{11} HZ) up to X-ray and gamma >10^{16} HZ .
- For geophysical Applications less than few thousand hertz, the wavelength of order 15-100 km , typical source- receiver separation is much smaller (4-10 m)



The primary EM field travels from the transmitter coil to the receiver coil via paths both above and below the surface.
In the presence of conducting body, the magnetic component of the EM field penetrating the ground induces alternating currents or eddy currents to flow in the conductor.
The eddy currents generate their own secondary EM field which travels to the receiver. Differences between TX and RX fields reveal the presence of the conductor and provide information on its geometry and electrical properties.

Depth of Penetration of EM

Skin Depth : is the depth at which the amplitude of a plane wave has decreased to 1/e or 37% relative to its initial amplitude $\mathbf{A_o}$.
Amplitude decreasing with depth due to absorption at two frequencies

$$\mathbf{A_z = A_o\, e^{-1}}$$

The skin depth S in meters = $\sqrt{2 / \omega\sigma\mu}$ = $503 \sqrt{f\,\sigma}$

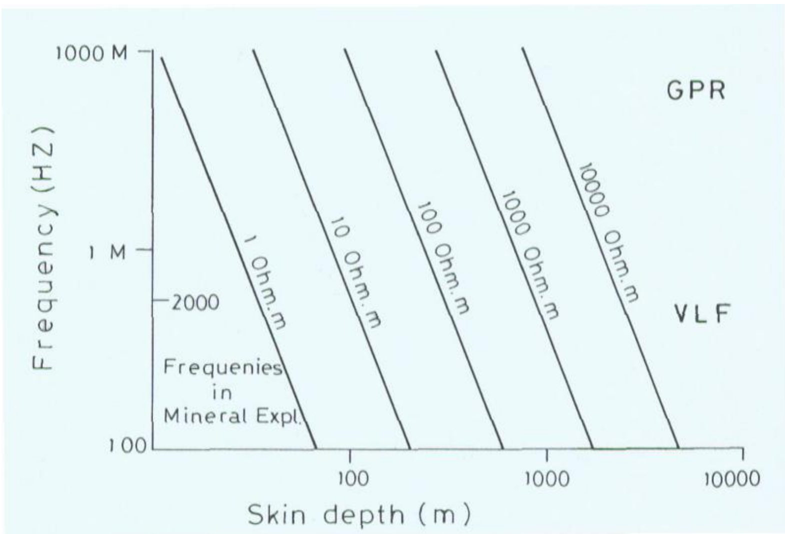
$\omega = 2\pi f$ = $503 \sqrt{\rho / f}$ = $503 \sqrt{\rho\,\lambda / v}$

σ : conductivity in s/m

μ : magnetic permeability (usually ≈ 1)

λ : wavelength , f : frequency , v : velocity , p : Resistivity thus, the depth increases as both frequency of EM field and conductivity decrease.

Ex. In dry glacial clays with conductivity $5\times 10^{-4}\ sm^{-1}$, S is about 225 m at a frequency of 10 KHZ .



Skin depths are shallower for both higher frequencies and higher conductivities (Lower resistivities).

Magnetotelluric Methods (MT)

Telluric methods: Faraday's Law of Induction: changing magnetic fields produce alternating currents. Changes in the Earth's magnetic field produce alternating electric currents just below the Earth's surface called Telluric currents. The lower the frequency of the current, the greater the depth of penetration. Telluric methods use these natural currents to detect resistivity differences which are then interpreted using procedures similar to resistivity methods.

MT uses measurements of both electric and magnetic components of The Natural Time-Variant Fields generated.
Major advantages of MT is its unique Capability for exploration to very great depths (hundreds of kilometers) as well as in shallow Investigations without using of an artificial power source .
Natural – Source MT uses the frequency range 10^{-3}-10 HZ , while audio – frequency MT (AMT or AFMAG) operates within $10\text{-}10^{4}$ HZ .

The main Application of MT in hydrocarbon Expl. and recently in meteoric impact, Environmental and geotechnical Applications.

$$\mathbf{P_a = 0.2 / f \ |E_x / B_y|^2 = 0.2 / f \ |E_x / H_y|^2 = 0.2 / f \ |Z|^2}$$

E_x (nv/km) , B_y , orthogonal electric and magnetic components.
B_y : magnetic flux density in nT .
H_y : magnetizing force (A/m) .
Z : cagniard impedance.

The changing magnetic fields of the Earth and the telluric currents they produce have different amplitudes. The ratio of the amplitudes can be used to determine the apparent resistivity to the greatest depth in the Earth to which energy of that frequency penetrates.

Typical equation:

apparent resistivity = $\frac{1}{5f}\left(\frac{E_x}{H_y}\right)^2$

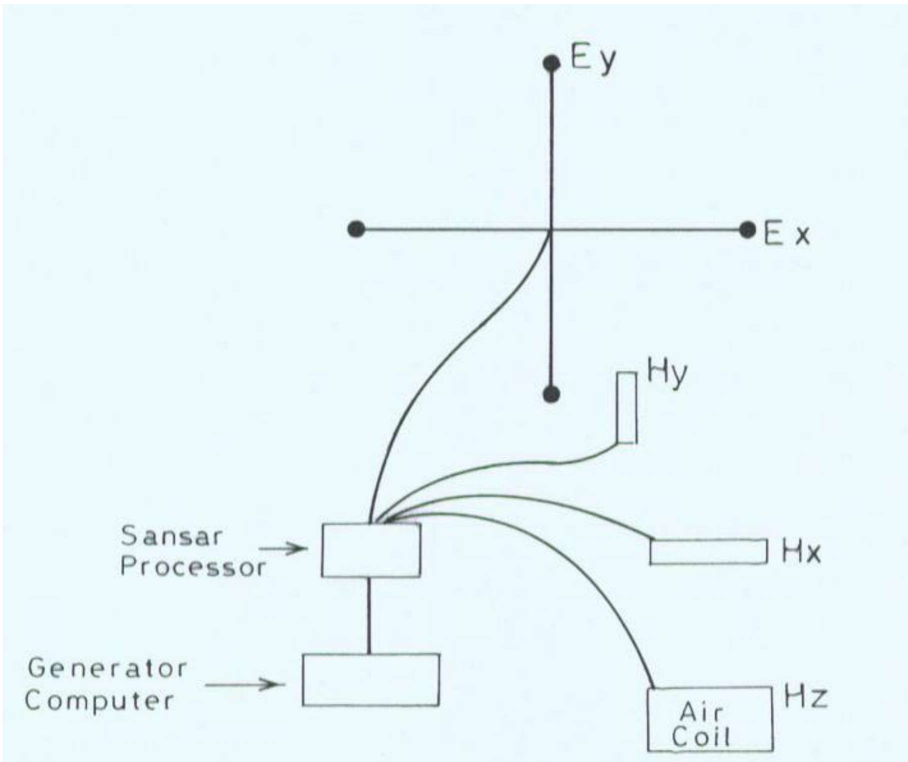
where E_x is the strength of the electric field in the x direction in millivolts H_y is the strength of the magnetic field in the y direction in gammas f is the frequency of the currents

$$\textbf{Depth of penetration} = \frac{1}{2\pi}\sqrt{\frac{5\ (\text{apparent resistivity})}{f}}$$

This methods is commonly used in determining the thickness of sedimentary basins. Depths are in kilometers

Field Procedure

MT Comprises two orthogonal electric dipoles to measure the two horizontal electric components and two magnetic sensors parallel to the electric dipoles to measure the corresponding magnetic components .



1. Two orthogonal grounded dipoles to measure electric components

2. Three orthogonal magnetic sensors to measure magnetic components.

Thus, at each location, five parameters are measured simultaneously as a function of frequency. By measuring the changes in magnetic (H) and electric (E) fields over a range of frequencies an apparent resistivity curve can be produced. The lower the frequency, the grater is the depth penetration.

Survey Design

EM data can be acquired in two configurations

1) Rectangular grid pattern
2) Along a traverse or profile .

EM equipment Operates in frequency domain. It allows measurement of both the .

1) in-phase (or real) component .
2) 90^0 out – of – phase (or quadrature) component.

Very Low Frequency (VLF) Method

VLF : uses navigation signal as Transmitter .
Measures tilt & phase

Main field is horizontal .

VLF detects electrical conductors by utilizing radio signal in the 15 to 30 KHZ range that are used for military communications.
VLF is useful for detecting long, straight electrical conductors
VLF compares the magnetic field of the primary signal (Transmitted) to that of the secondary signal (induced current flow with in the subsurface electrical conductor).

Advantages of VLF

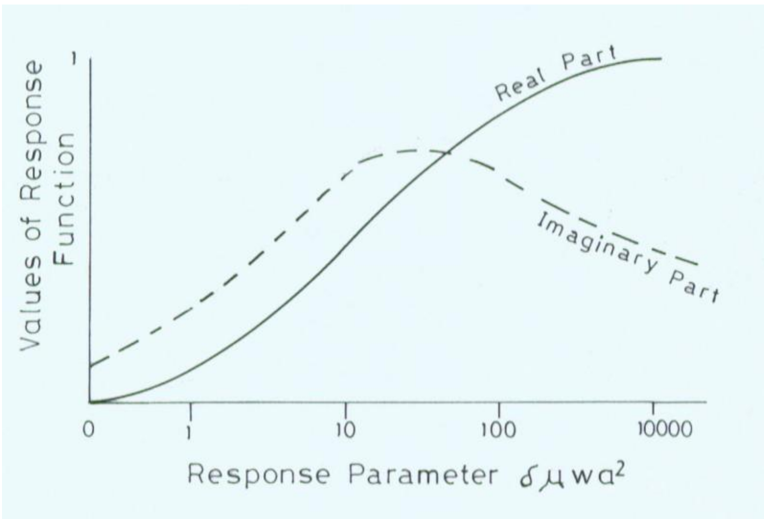
1) Very effective for locating zones of high electrical conductivity
2) fast
3) inexpensive
4) Requires one or two people .

Tilt Angle Method

Tilt angle systems have no reference link between Tx and Rx coils . Rx measures the total field irrespective of phase and the receiver coil tilted to direction of maximum or minimum magnetic field strength .

The response parameter of a conductor is defined are the product of conductivity – thickness (T) , permeability (μ) an angular frequency
$\omega = 2\pi f$ and the square of the target a^2 .

Poor conductors have response parameter < 1
Excellent conductor have response parameter greater than 1000



A Good conductor having a higher ratio A_R / A_i
A_R : Amplitude of Read (in – phase)
A_i : Amplitude of imaginary (out – of phase)

In the left side of the above figure and poor conductor having a lower ratio of A_R/ A_i .

Slingram System

- slingram is limited in the size of TX coil. This system has the Transmitter and Receiver connected by a cable and their separation kept constant as they are moved together along a traverse.

Magnetic field Through The receiver has two sources :

a) The primary field of The Transmitter .
b) The secondary field produced by The Target .

Turam system

More powerful system than Slingram. It uses a very large stationary Transmitter coil or wire laid out on the ground, and only The receiver is moved . TX 1-2 km long, loop over 10 km long. The receiver consists of two coils and kept a fixed distance between 10-50 m apart.

Ground Surveys of EM

A. Amplitude measurement

1- Long wire

- Receiver pick up horizontal component of field parallel to wire .
- Distortions of Normal field pattern are related to changes in subsurface conductivity.

B. Dip-Angle

Measures combined effect of primary and secondary fields at the receiver.

AFMAG : Dip-angle method that uses Naturally occurring ELF signals generated by Thunder storms.

Phase Component Methods

1) Work by comparing secondary & primary fields .
2) Compensator & Turam (long wire) .
3) Slingram Moving Trans / receiver

- Penetration ≈ ½ spacing of coil .
- Coil spacing critical .

- Over barren ground Null is at zero coil dip-Angle.
- Near conductor, dip angle $\neq 0$
- Dip – Angle is zero over Narrow conductor, and changes sign.

Dip-Angle method

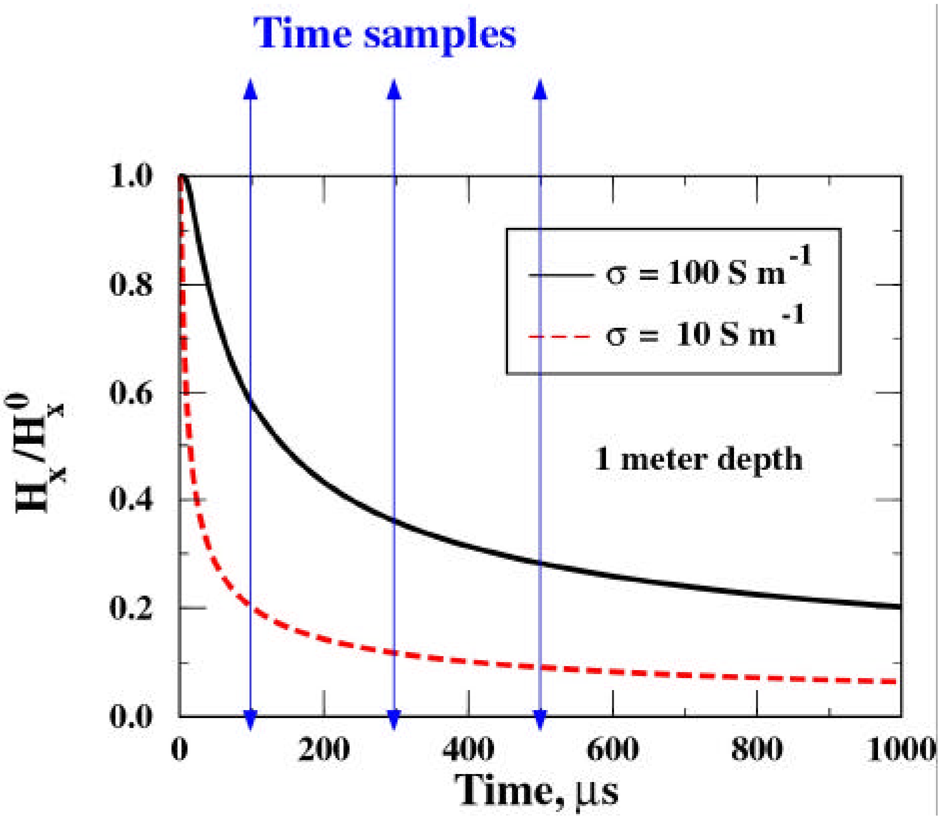
1) easy , cheap
2) Quick
3) Sensitive to vertical
4) Difficult to distinguish between depth & conductivity

TDEM Method

A significant problem with many EM surveying techniques is that a small secondary field must be measured in the presence of a much larger primary field, with a consequent decrease in accuracy. This is circumvented to some extent in the FDEM method described about by measuring the out-of-phase component.

In a TDEM approach, the signal is not a continuous frequency but instead consists by a series of step-like pulses separated by periods where there is no signal generated, but the decay of the secondary field from the ground is measured. The induction

currents induced in a subsurface conductor diffuse outward when the inducing energy is suddenly switched off. The measurement of the field at a number of time steps is equivalent of measuring at several frequencies in an FDEM system. Usually two-coil systems are used and the results can be stacked to reduce noise. Modeling of the decay for layered systems, and more complicated conductivity geometry, can be carried out.



The figure above shows the behavior of the field for two different conductivities. If samples at different times were taken, then on could distinguish between the two conductivities. This is the principle of Time Domain Electromagnetic (TDEM) methods.

$\boldsymbol{\tau = \sigma \mu L^2 = \sigma \mu A}$

where τ is a characteristic time constant and L and A correspond to a characteristic length scale and characteristic area, respectively.

The EM61 has a single time sample at $t \approx 0.5$ ms. Using a cylinder of radius 2 cm and a conductivity of steel of 10^7 S m^{-1}, then $\exp(-t/\tau) = 0.56$. where t is time constant.

On the other hand, assume a plastic drum of seawater of conductivity 10 S m^{-1} and radius 40 cm, then we obtain $\exp(-t/\tau) = 0$.

Airborne Electromagnetic Surveys

The general objective of **AEM** (Airborne ElectroMagnetic) surveys is to conduct a rapid and relatively low-cost search for metallic conductors, e.g. massive sulphides, located in bed-rock and often under a cover of overburden and/or fresh water. This method can be applied in most geological environments except where the country rock is highly conductive or where overburden is both thick and conductive. It is equally well suited and applied to general geologic mapping, as well as to a variety of engineering problems (e.g., fresh water exploration.) Semi-arid areas, particularly with internal drainage, are usually poor AEM environments.

Conductivities of geological materials range over seven orders of magnitude, with the strongest EM responses coming from massive sulphides, followed in decreasing order of intensity by graphite, unconsolidated sediments (clay, tills, and gravel/sand), and igneous and metamorphic rocks. Consolidated sedimentary rocks can range in conductivity from the level of graphite (e.g. shales) down to less than the most resistive igneous materials (e.g. dolomites and limestones). Fresh water is highly resistive. However, when contaminated by decay material, such lake bottom sediments, swamps, etc., it may display conductivity roughly equivalent to clay and salt water to graphite and sulphides.

Typically, graphite, pyrite and or pyrrhotite are responsible for the observed bedrock AEM responses. The following examples suggest possible target types and we have indicate the grade of the AEM response that can be expected from these targets.

- Massive volcano-sedimentary stratabound sulphide ores of Cu, Pb, Zn, (and precious metals), usually with pyrite and/or pyrrhotite. Fair to good AEM targets accounting for the majority of AEM surveys.
- Carbonate-hosted Pb-Zn, often with marcasite, pyrite, or pyrrhotite, and sometimes associated with graphitic horizons. Fair to poor AEM targets.
- Massive pyrrhotite-pentlandite bodies containing Ni and sometimes Cu and precious metals associated with noritic or other mafic/ultramafic intrusive rocks. Fair to good AEM targets.
- Vein deposits of Ag, often with Sb, Cu, Co, Ni, and pyrite in volcanic and sedimentary rocks. Generally poor AEM targets.
- Quartz veins containing Au with pyrite, sometimes also with Sb, Ag, Bi, etc., in volcanic or sedimentary (and possibly intrusive) rocks. Poor AEM targets.

Basic Principles of Airborne

Electromagnetic-induction prospecting methods, both airborne and (most) ground techniques, make use of man-made primary electromagnetic fields in, roughly, the following way: An alternating magnetic field is established by passing a current through a coil, (or along a long wire). The field is measured with a receiver consisting of a sensitive electronic amplifier and meter or potentiometer bridge. The frequency of the alternating current is chosen such that an insignificant eddy-current field is induced in the ground if it has an average electrical conductivity,

If the source and receiver are brought near a more conductive zone, stronger eddy currents may be caused to circulate within it and an appreciable secondary magnetic field will thereby be created. Close to the conductor, this secondary or anomalous field may be compared in magnitude to the primary or normal field (which prevails in the absence of conductors), in which case it can be detected by the receiver. The secondary field strength, Hs, is usually measured as a proportion of the primary field strength, Hp, at the receiver in percent or ppm (parts per million).

Anomaly = Hs / Hp.

Increasing the primary field strength increases the secondary field strength proportionally but the "anomaly" measured in ppm or percent remains the same.
prospecting for anomalous zones is carried out by systematically traversing the ground either with the receiver alone or with the source and receiver in combination, depending on the system in use. In the case of airborne systems, the receiver coils are usually in a towed bird and the transmitter may be a large coil encircling a fixed wing aircraft, e.g. INPUT systems, or one or more small coils in the same bird that houses the transmitting coils, e.g. most HEM (Helicopter EM) systems.
There are two different basic systems commonly used to generate and receive the electromagnetic field: transient or "time domain" systems like INPUT,GEOTEM and MEGATEM and a/c. "frequency domain" systems like most HEM systems.

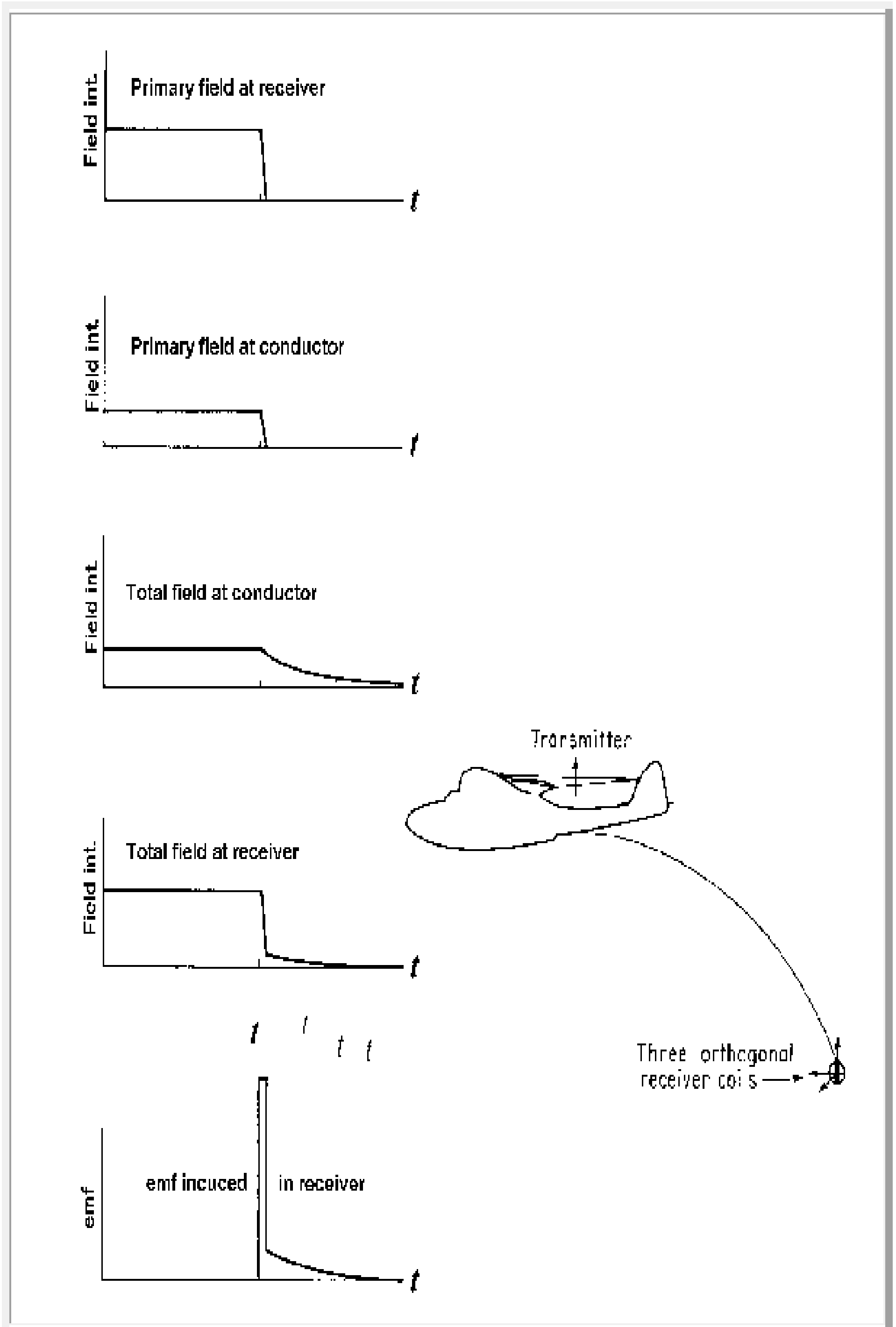
Transient Airborne Electromagnetics

Historically, the most commonly encountered system of this type was the INPUT system. The newer systems GEOTEM and MEGATEM (Fugro Airborne Surveys) function in a similar way to INPUT

In the INPUT system the transmitting coil, usually encircling a fixed wing aircraft, is energized by what is, essentially, a step current. In the absence of conductors, a sharp transient pulse proportional to the time derivative of the magnetic field is induced in the receiver. When a conductor is present, however, a sudden change in magnetic field intensity will induce in it a flow of current in the conductor which will tend to slow the decay of the field.

The receiver "listens" only while the transmitter is "quiet" so that problems arising out of relative motion between transmitter and receiver, because the receiver is towed in a bird behind the aircraft, are virtually eliminated. Moreover, if the entire decay of the secondary field could be observed, the response would be equivalent to AC measurements made over the whole of the frequency spectrum. It is important to note in this connection, however, that not the decay function itself but only its time derivative can be recorded if a coil is used as the detector. This means that the anomalous fields which decay very slowly are suppressed in amplitude more than the others, and since these are the very ones generally associated with good conductors, there would seem to be an inherent weakness in this system. Because it is difficult to precisely synchronize the instant when the transmitter becomes "quiet" with the instant that the receiver begins to "listen", it is nearly impossible to record the entire function. This is equivalent to being unable to record many of the lower frequencies in the a-c spectrum.

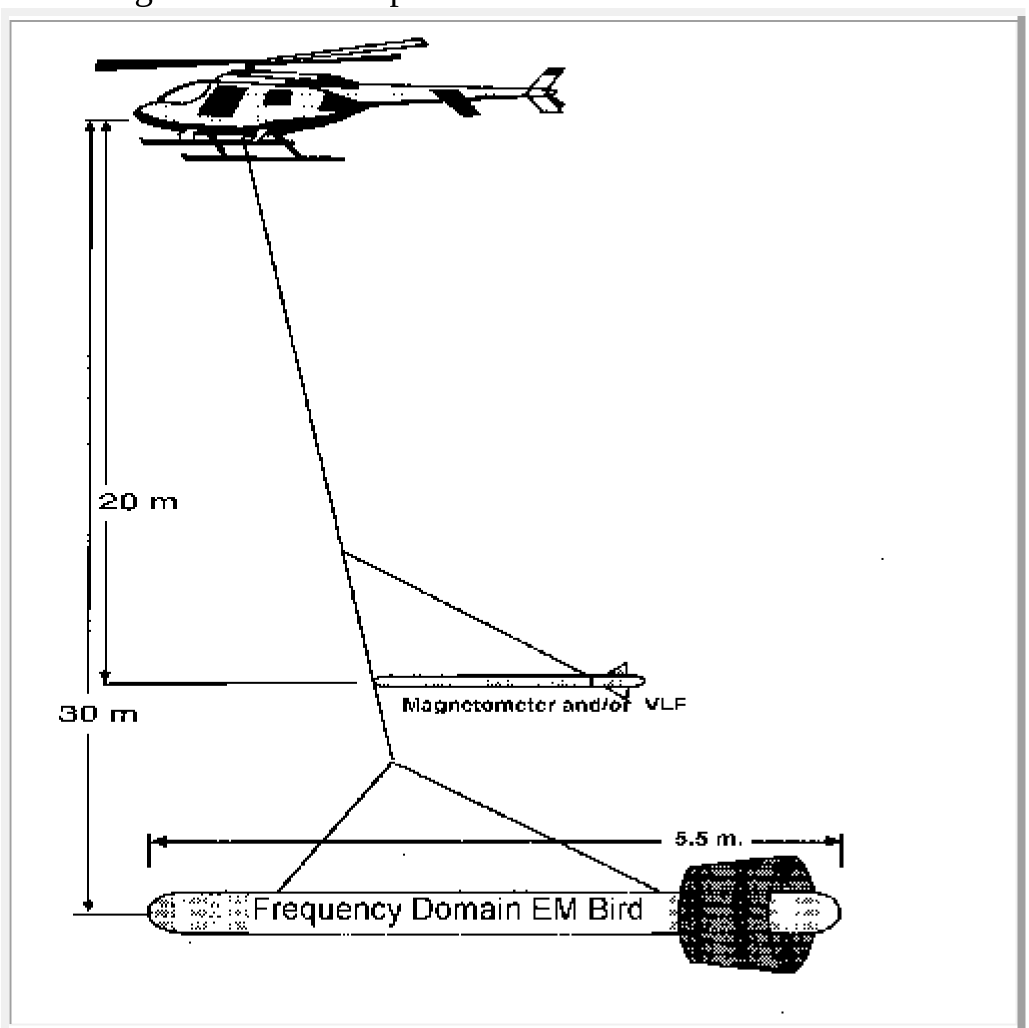
Typically, the time derivative of the decay function is measured using from six to twelve different time delays from the instant that transmitter stops transmitting before recording the signal received.



A sketch of the INPUT transient airborne EM system operation. The primary field is a step function and the receiver records the decay of the field after the transmitter stops transmitting. (Grant and West 1965)

Frequency Domain Airborne Electromagnetics

In the typical frequency domain helicopter EM system (HEM) both the transmitting coil set and the receiver coil set are housed in a rigid boom or "bird" that is towed beneath the helicopter. Commonly, this boom is from three to five meters long and contains from two to six coil pairs. Usually, half of the coils in each of the transmitter set and the receiver set are "co-axial", i.e. an axis normal to the plane of the coils passes through the centre of both coils. The second half of the coil sets are normally "co-planar", being equivalent to both the transmitting and receiving coil lying flat on the ground. Other coplanar orientations have been used occasionally.



The receiver measures the in-phase and out-of-phase, or quadrature, of the secondary field, expressed in ppm of the primary field. The two different coil orientations provide data that is useful in discriminating between dike like conductors that have considerable vertical extent and may be ore bodies, and horizontal sheet like conductors that are simply conductive overburden.

Factors Affecting Detectability

1. Signal-to-noise ratio:

In practice, because of "system noise" (Ns) and "geological noise" (Ng), the ability of a system to recognize and measure an anomaly is limited by the "signal-to-noise" ratio: **Signal-to-noise = Hs / (Ns + Ng)**
Because Hs and Ng are proportional to the primary field strength Hp, and Ns, in frequency-domain systems, usually contains elements proportional to Hp, there is little to be gained by increasing the primary field power. In time domain systems Ns is not greatly affected by Hp, so extra power does result in increased signal-to-noise. Attempts to increase the signal-to-noise are sometimes made by increasing the distance between the transmitter and receiver. This results in roughly the same Hs and Ng but often a lower system noise Ns.

2. Penetration

The penetration of an AEM system is its effective depth of exploration. Commonly, this is taken to include the elevation of the system above ground, as this is also affected by local environment and flying conditions.
In general, systems with large transmitter-receiver coil separation, usually referred to as Tx-Rx, have greater penetration than those with small separations. Penetration is closely related to signal-to-noise, as the system that produces a larger anomaly from a given conductor can, of course, look further into the ground.

3. Discrimination

The discrimination of an AEM system is the ability of the system to differentiate between conductors of different physical properties or geometric shapes. Discrimination, particularly between flat lying surficial conductors and steeply dipping conductors, is vitally important. Good discrimination can be achieved in HEM systems by using several frequencies and both co-axial and co-planar coil pairs.

4. Resolution

Resolution refers to the ability of an AEM system to recognize and separate the interfering effects of nearby conductors. A system that does this well also produces sharp anomalies over isolated or discrete conductors. Resolution generally increases with decreasing flight elevation and coil separation. Typically the HEM systems have better resolution than the fixed wing time domain systems.

Typical Electrical Properties of Earth Materials.

Rock, Mineral, etc.	Conductivity (mohs/meter)	Resistivity (ohm-meters)
Bornite	330	3×10^{-3}
Chalcocite	10^{4}	10^{-4}
Chalcopyrite	250	4×10^{-3}
Galena	500	2×10^{-3}
Graphite	10^{3}	10^{-3}
Marcasite	20	5×10^{-2}
Magnetite	$17 \times 10^{-4} - 2 \times 10^{4}$	$5 \times 10^{-5} - 6 \times 10^{-3}$
Pyrite	3	0.3
Phrrhotite	10^{4}	10^{-4}
Sphalerite	10^{-2}	10^{2}
Igneous and Metamorphic Rocks	$10^{-7} - 10^{-2}$	$100 - 10^{7}$
Sediments	$10^{-5} - 5 \times 10^{-2}$	$20 - 10^{5}$
Soils	$10^{-3} - 0.5$	$2 - 10^{3}$
Fresh Water	$5 \times 10^{-3} - 0.1$	10 - 200
Saline Overburden	0.1 - 5	0.2 - 1
Salt Water	5 - 20	0.05 - 2
Sulphide Ores	$10^{-2} - 10$	0.1 - 100
Granite Beds and Slates	$10^{-2} - 1$	1 - 100
Altered Ultramafics	$10^{-3} - 0.8$	$1.25 - 10^{3}$
Water-filled faults/shears	$10^{-3} - 1$	$1 - 10^{3}$

Ground penetrating radar (GPR)

Ground penetrating radar is a nondestructive geophysical method that produces a continuous cross-sectional profile or record of subsurface features, without drilling, probing, or digging. Ground penetrating radar (GPR) profiles are used for evaluating the location and depth of buried objects and to investigate the presence and continuity of natural subsurface conditions and features.

Ground penetrating radar operates by transmitting pulses of ultra high frequency radio waves (microwave electromagnetic energy) down into the ground through a transducer or antenna. The transmitted energy is reflected from various buried objects or distinct contacts between different earth materials. The antenna then receives the reflected waves and stores them in the digital control unit.

The ground penetrating radar antenna (transducer) is pulled along the ground by hand or behind a vehicle.

When the transmitted signal enters the ground, it contacts objects or subsurface strata with different electrical conductivities and dielectric constants. Part of the ground penetrating radar waves reflect off of the object or interface; while the rest of the waves pass through to the next interface.

The reflected signals return to the antenna, pass through the antenna, and are received by the digital control unit. The control unit registers the reflections against two-way travel time in nanoseconds and then amplifies the signals. The output signal voltage peaks are plotted on the ground penetrating radar profile as different color bands by the digital control unit.

For each reflected wave, the radar signal changes polarity twice. These polarity changes produce three bands on the radar profile for each interface contacted by the radar wave.

Ground penetrating radar waves can reach depths up to 100 feet (30 meters) in low conductivity materials such as dry sand or granite. Clays, shale, and other high conductivity materials, may attenuate or absorb GPR signals, greatly decreasing the depth of penetration to 3 feet (1 meter) or less.

The depth of penetration is also determined by the GPR antenna used. Antennas with low frequencies of from 25 to 200 MHz obtain subsurface reflections from deeper depths (about 30 to 100 feet or more), but have low resolution. These low frequency antennas are used for investigating the geology of a site, such as for locating sinkholes or fractures, and to locate large, deep buried objects.

Antennas with higher frequencies of from 300 to 1,000 MHz obtain reflections from shallow depths (0 to about 30 feet), and have high resolution. These high frequency antennas are used to investigate surface soils and to locate small or large, shallow buried objects and rebar in concrete.

GPR is a high-resolution technique of imaging shallow soil and ground structures using electro-magnetic (EM) waves in the frequency band of 10-1000 MHz. Advantage of EM waves is that signals of relatively short wavelength can be generated and radiated into the ground to detect anomalous variations in dielectric properties of the geological material

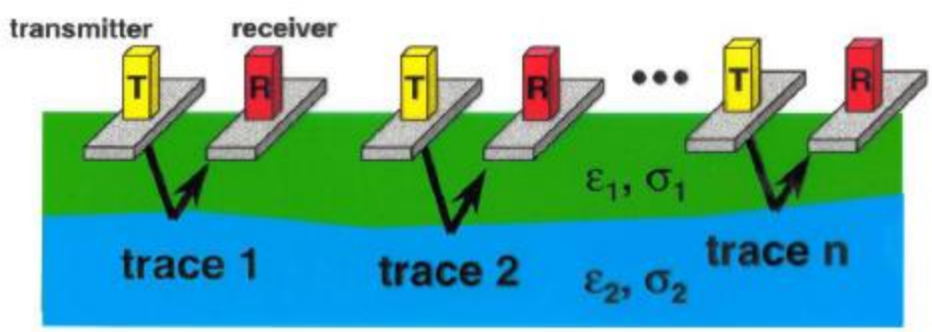
GPR – technical summary

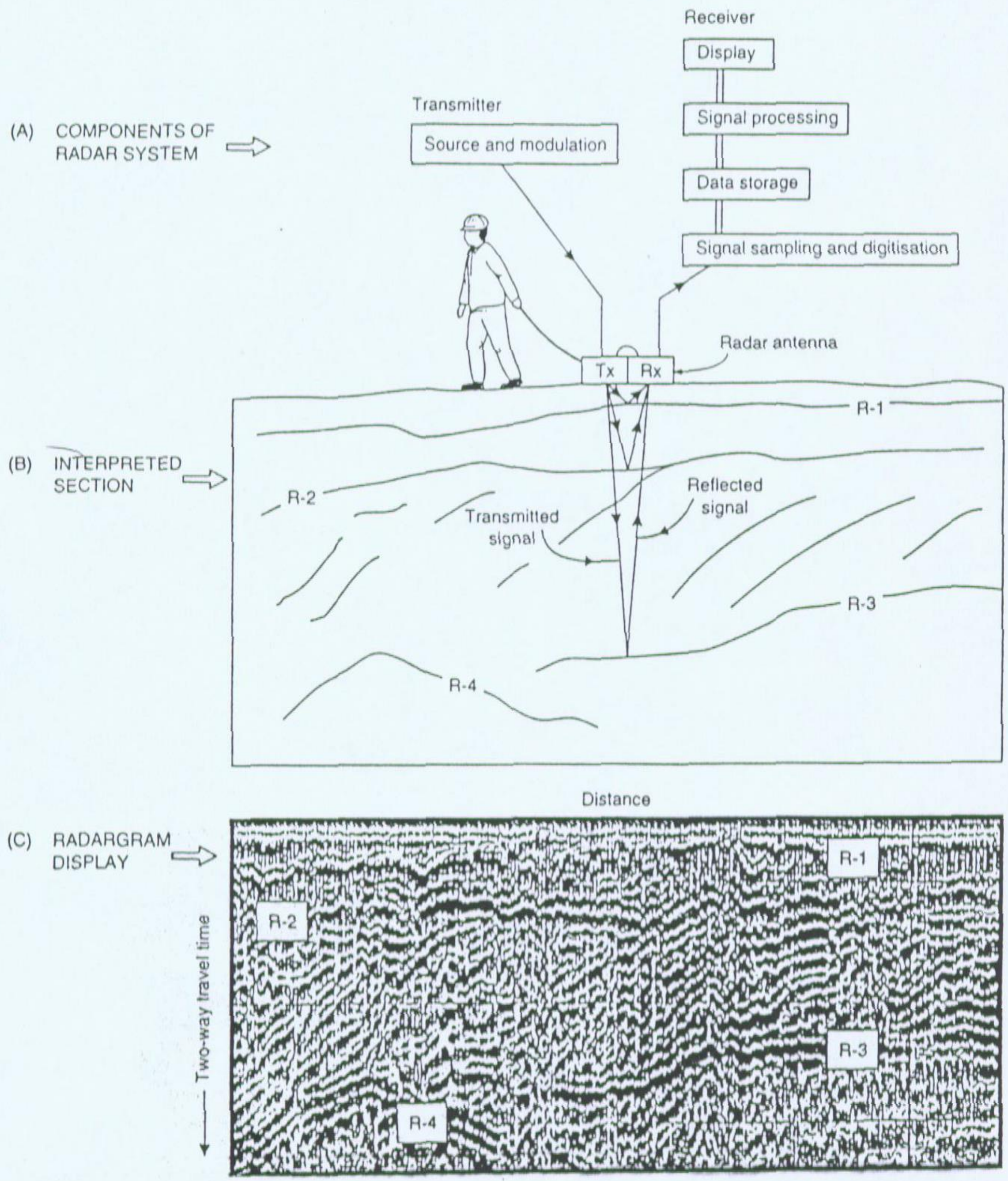
- f ~ 50 MHz - 5 GHz
- v ~ 0.05 - 0.15 m/ns
- λ ~ 1 m - 1cm
- Reach : 0 - 50 λ

Used on surface with one sensor for transmitting and receiving or double system of separate transmitter/receiver Used in boreholes (3D borehole radar, or cross imaging)

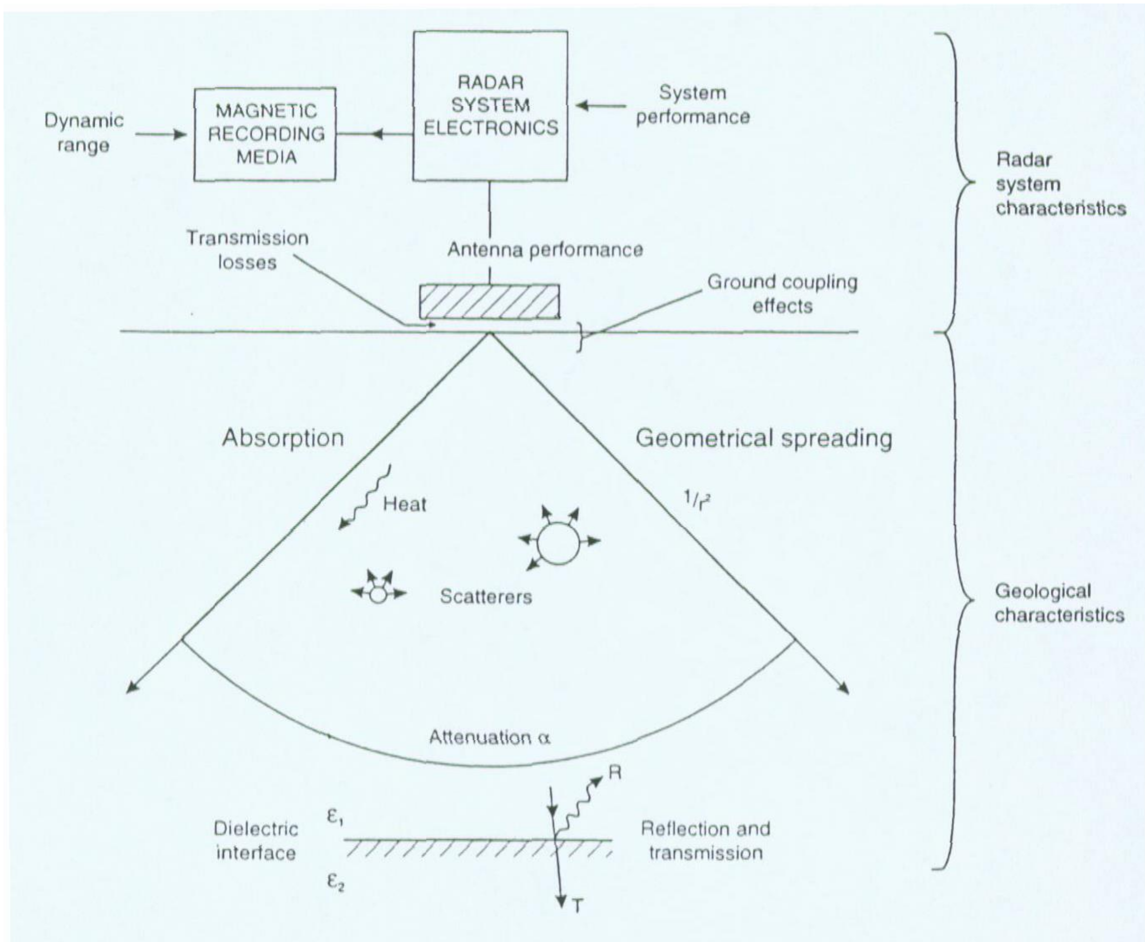
Principles of operation

A radar system comprises a:
signal generator
transmitting / receiving antennae
recording unit





(A) COMPONENTS OF RADAR SYSTEM
Receiver
Display
Signal processing
Data storage
Signal sampling and digitisation
Transmitter
Source and modulation
Radar antenna
Tx
Rx
(B) INTERPRETED SECTION
R-1
R-2
R-3
R-4
Transmitted signal
Reflected signal
Distance
(C) RADARGRAM DISPLAY
Two-way travel time
R-1
R-2
R-3
R-4



Factors Controlling Loss of Energy

GPR - application

- Geological
- Environmental
- Glaciological
- Engineering and construction
- Archaeology
- Forensic science

Geological

- Detection natural cavities and fissures
- Subsidence mapping
- Mapping of superficial deposits
- Soil stratigraphy mapping
- Geological structure mapping
- Mapping of faults, dykes, coal seams
- Lake and riverbed sediment mapping
- Mineral exploration and resource evaluation

Engineering and construction

- Road pavement analysis
- Void detection
- Location of reinforcement (rebars) in concrete
- Location of public utilities (pipes, cables, etc)
- Testing integrity of building materials
- Concrete testing

Dielectric constant of radar waves

Material	ε_r
Dry sand/gravel	4-10
Wet sand/gravel	10-20
Dry clay/silt	3-6
Wet clay/silt	7-40
Granite	4-9
Limestone	4-8
Dry salt	5-6
Permafrost	4-5
Glacier ice	3.5
Fresh water	81
Kerosene	2.1

What does GPR measure?

GPR systems essentially measure the signal travel time, the time between sending the pulse at the transmitter antenna and the moment the (distorted) pulse is received back at the receiver antenna.
In general one measures not only one peak but a series of peaks related to various objects in the sub-surface. The inhomogeneity of the sub-surface is the main factor that controls the number of peaks that will be returned.

Resolution

- In practice, it is often better to accept lower spatial resolution in favour of range where there are many thin layers or scattering targets that are not of primary interest.

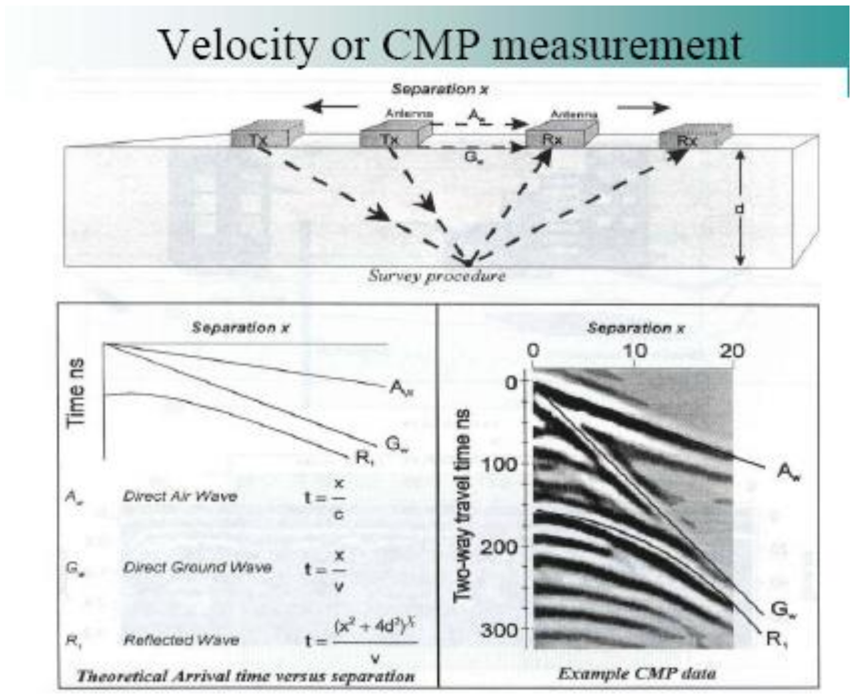
- Low frequency GPR
  1. Deep penetration
  2. Low resolution

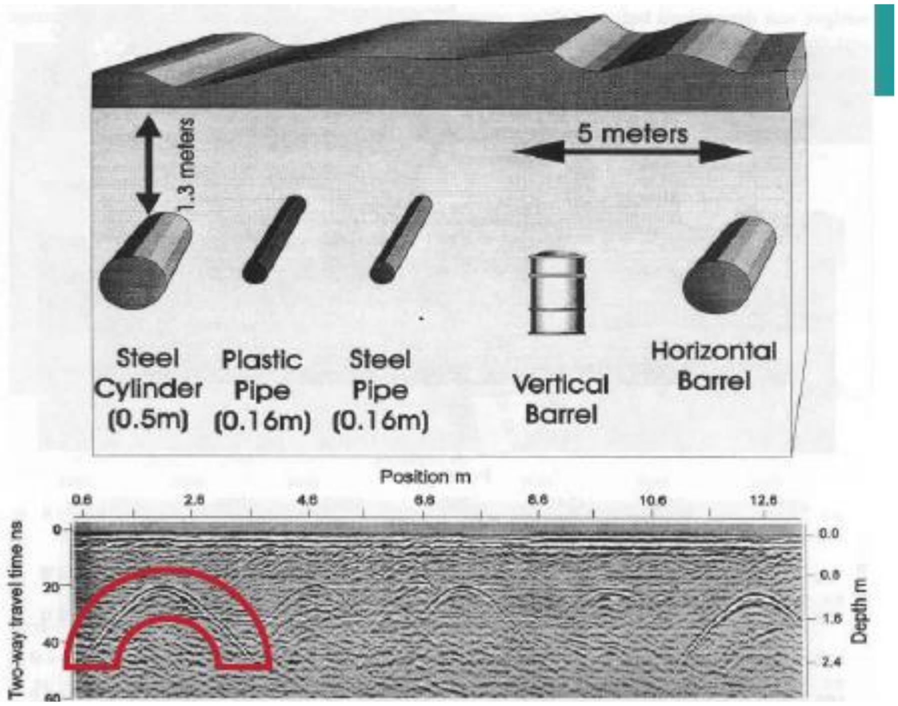
- High frequency GPR
  1. Shallow penetration
  2. Very high resolution

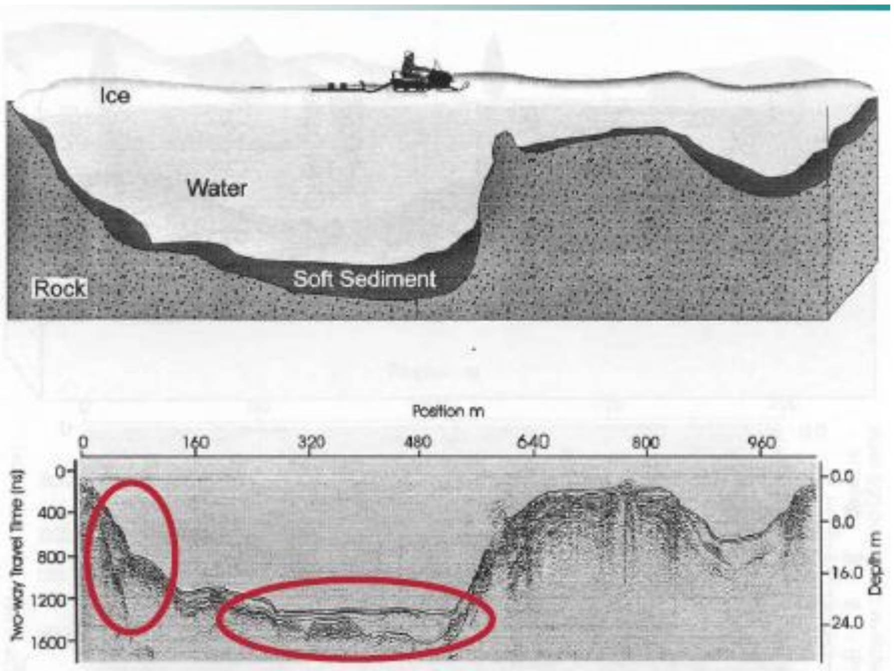
Data processing

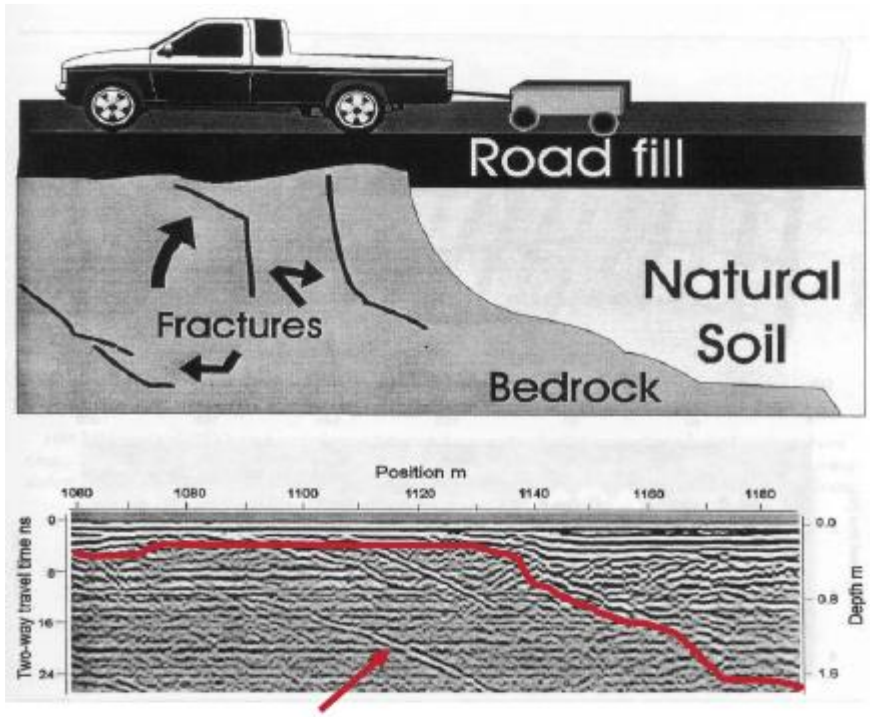
Data processing for GPR is very similar to that used in seismics. Most seismic software packages can deal with radar measurements.

Time – depth conversions can be made through estimation of velocity. There are various ways to do this. CMP measurements is just one possibility.

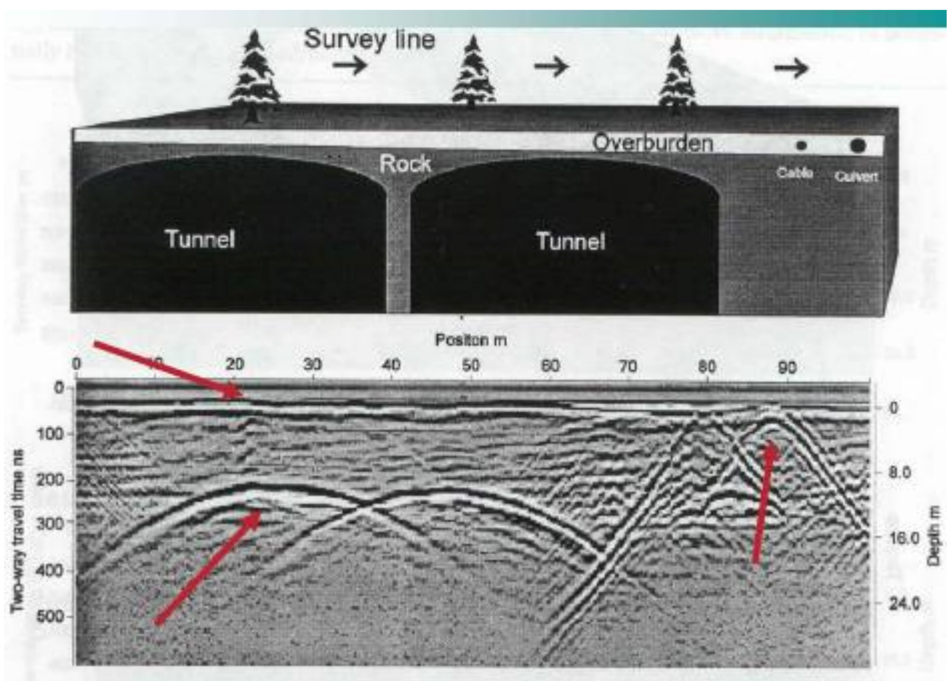



1.3 meters
5 meters
Steel
Cylinder
(0.5m)
Plastic
Pipe
(0.16m)
Steel
Pipe
(0.16m)
Vertical
Barrel
Horizontal
Barrel
Position m
Two-way travel time ns
Depth m

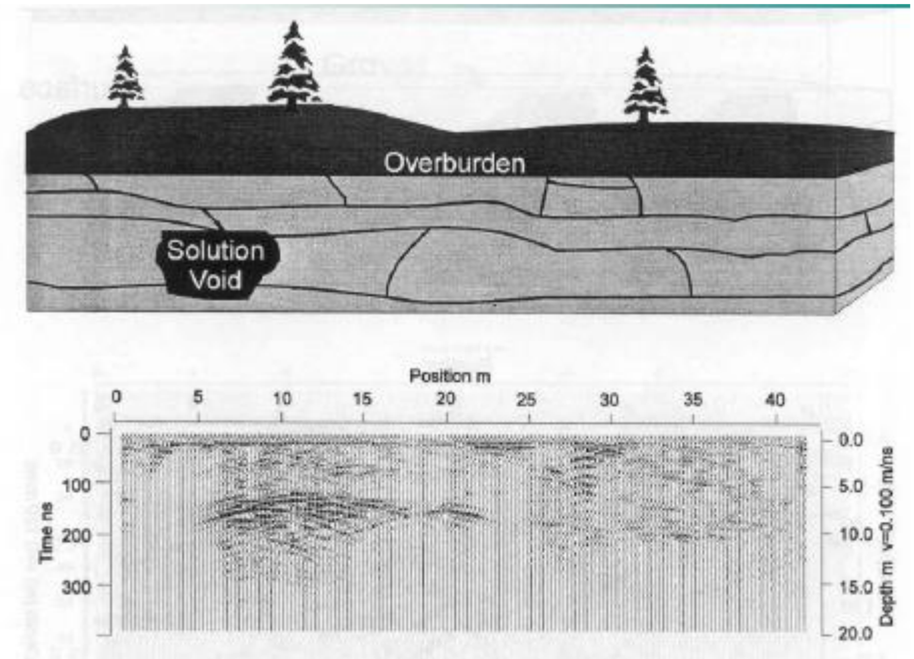

Ice
Water
Rock
Soft Sediment
Position m
0
160
320
480
640
800
960
Two-way Travel Time (ns)
0
400
800
1200
1600
Depth m
0.0
8.0
16.0
24.0

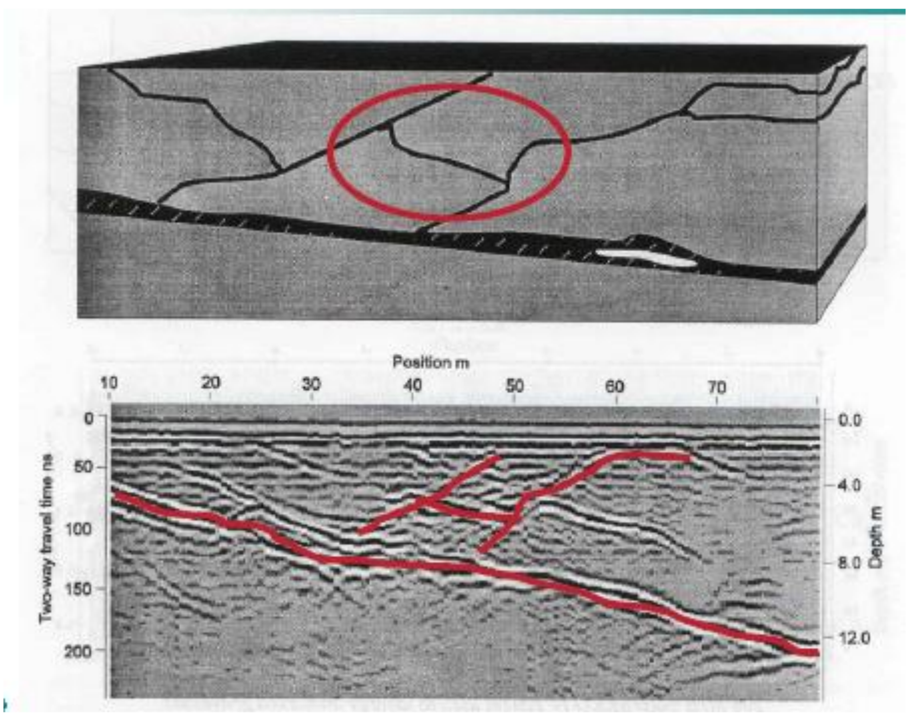


Road fill
Natural
Soil
Fractures
Bedrock
Position m
1060
1080
1100
1120
1140
1160
1180
Two-way travel time ns
Depth m



Survey line
Overburden
Rock
Cable
Culvert
Tunnel
Tunnel
Positon m
Two-way travel time ns
Depth m


Overburden
Solution Void
Position m
0 5 10 15 20 25 30 35 40
Time ns
0 100 200 300
Depth m v=0.100 m/ns
0.0 5.0 10.0 15.0 20.0


Position m
10 20 30 40 50 60 70
Two-way travel time ns
0 50 100 150 200
Depth m
0.0 4.0 8.0 12.0

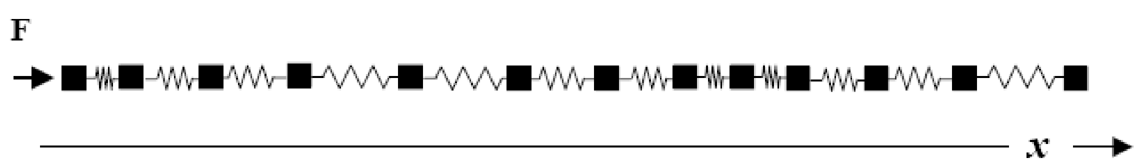
SEISMIC METHODS

Seismic Waves :

1. Waves in a mass- spring system

A simple physical model that is useful for describing the motion of a sound wave in a gas or fluid is shown in the following cartoon. The density of the medium is represented by the distributed masses, and the compressibility is represented by the distributed springs. The masses and springs should be envisioned as lying in a frictionless 'track' which confines any motion to the x coordinate direction.

In this thought experiment the first mass is displaced to the right by an external impulsive force F. The mass accelerates to the right, compressing the spring and transmitting the force to the second mass. Meanwhile the external force is removed and the compressed first spring returns the first

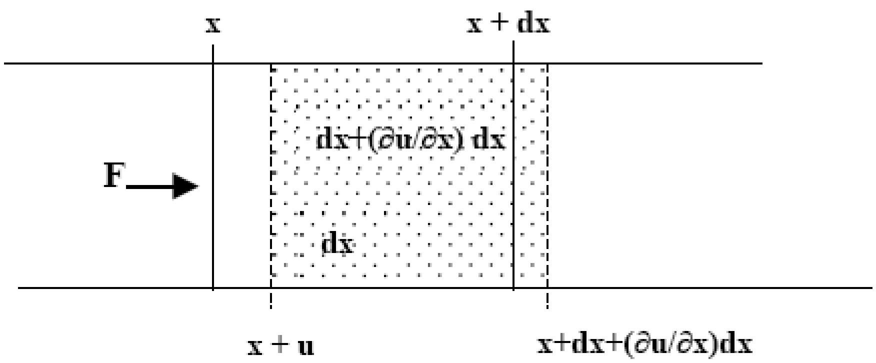


mass towards its initial position while acting on the second mass to transmit the forces to the right essentially initiating a continuous sequence of mass displacements along the chain of masses. Examined in time, any individual mass would be observed to undergo a short oscillatory movement in the x direction as the disturbance moved along the chain. A snapshot in time would show a chain of compressions and extensions of the springs as in the above cartoon. Successive snapshots in time of the whole chain would show the disturbance progressing from left to right as a wave. A particular state of, say, compression, called a point of constant phase of the wave, propagates to the right with the phase velocity, V_{ph}.

2. Acoustic waves in a liquid or gas

A quantitative analysis of this phenomenon in a liquid illustrates the relationship between the wave and the density and compressibility of the liquid or gas.

Consider two planes at x and x + dx perpendicular to the x axis with a uniform force applied in the x direction on the plane at x. A small patch on these planes has area A. The volume A dx is an *element* of the medium.



The force applied to the plane at x causes its displacement to x + u which in turn causes the second plane to be displaced by a different amount given by the rate of change of the displacement with x, times the separation of the planes or $(\partial u/\partial x)dx$. The strain, the change in dimension in the x direction normalized by the unstressed dimension in x, is then simply $\partial u/\partial x$.
The stress at x is P, which is the force per unit area or F/A. The force also varies with x so we may write the stress at x + dx as $P + (\partial P/\partial x)\, dx$.

Hooke's law relates the stress and strain through the elastic constant of the medium, in this case through the bulk modulus K. So writing the stress at x in terms of K and the above strain we find that:

$P = K \times \text{strain} = K\,(\partial u/\partial x)$

P changes with x and so at $x = x + dx$,

$P = P + (\partial P/\partial x)\, dx$

$= P + \partial/\partial x\,(K\,(\partial u/\partial x)\, dx$

$= P + K\,(\partial^2 u/\partial x^2)\, dx$

So there can be an unbalanced force, $A\,K(\partial^2 u/\partial x^2)\, dx$, acting on the element dx which causes it to accelerate. By Newton's Law this force must equal the mass of the element times its resulting acceleration, so

$A\,K\,(\partial^2 u/\partial x^2)\, dx = \rho\, A\, dx\, \partial^2 u/\partial t^2$

where ρ is the density of the medium.

So, finally, we have an equation of motion for the element,

$\partial^2 u/\partial x^2 - \rho/K\, \partial^2 u/\partial t^2 = 0$

Since we normally deal with oscillatory or quasi-harmonic disturbances we can explore the behavior of solutions to this equation by assuming that the solution in time is sinusoidal and that the time dependence is given by $e^{i\omega t}$,

where ω is the angular frequency in radians/sec (equal to 2πf, where f is the frequency in cycles per second or Hertz). With this solution in t the equation of motion becomes:

$\partial^2 u/\partial x^2 + \omega^2 \rho/K\, u = 0$

or, with $\omega^2 \rho/K = k^2$

$\partial^2 u/\partial x^2 + k^2 u = 0$

The solution for the displacement u is,

$u(x,t) = u_0 e^{i\omega t} e^{\pm ikx} = u_0 e^{i(\omega t \pm kx)}$

This solution is basically the definition of a propagating wave. At a fixed position in x, a particular element of the medium oscillates back and forth in the x direction about a rest position (corresponding to the in-line oscillation of a particular mass in the above mass-spring analog). At a fixed time the displacements are distributed sinusoidally along x just as the spring distortions were distributed in the mass-spring analog.

Another way to visualize the solution is to imagine the movement of a molecule or particle in the liquid. The particle movement would be back and forth along the x-axis, an oscillatory movement in the direction of propagation. At this microscopic level the masses in the mass-spring analog are like particles in the liquid.

The derivation for compressive-extensional waves in a thin rod is identical except that the bulk modulus is replaced by Young's modulus, E.

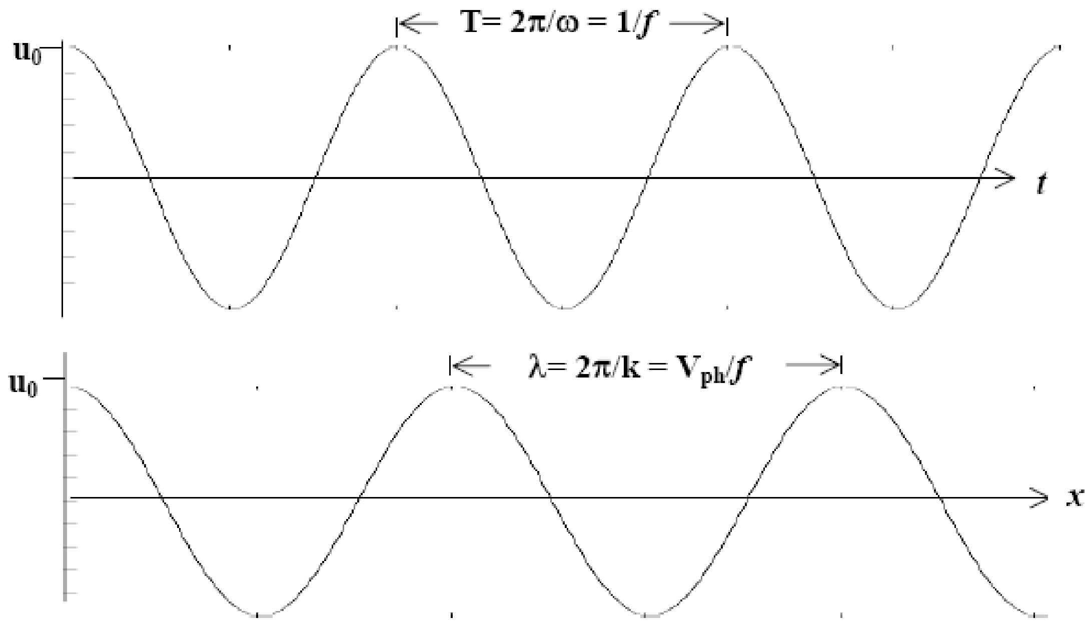
The exponent ωt ± kz is called the phase and we can easily see what is required to stay at a point in x and t where the phase is constant. When ωt ± kz = C, ∂z/∂t = ±ω/k which is the velocity of the phase point, the phase velocity V_{ph}. The plus sign is for a wave traveling in the plus x direction, the minus sign for one traveling in the opposite direction.

Substituting for k, the phase velocity is given by:

$V_{ph} = (K/\rho)^{1/2}$

The phase velocity is equal to the square root of the ratio of the bulk modulus to the density. Denser fluids thus have lower velocity than lighter fluids. Gases follow the

same formula. The table below lists some densities and compressibilities and the derived velocities for some common liquids and air.
The distance at a given time between two successive points of equal phase on the wave is the wavelength, λ, and the time between two successive points of equal displacement at a given position on the x axis is the period, T, of the wave.
The velocity, frequency, period and wavelength are all related as can be seen in the following figure.



3. Seismic waves in a solid

The derivation of the equation of motion for an element in a solid is basically the same as that outlined above but it is much more complicated to the fact that the restoring elastic forces include shear as well as compression. The detailed analysis, done very well for example in Telford et al., shows that there exists a compressional wave with particle motion in the direction of wave propagation and also a shear wave with particle motion in the plane perpendicular to the direction of propagation. The velocities of these waves are given in terms of the bulk and shear modulii, K, μ and density, ρ, viz.:

$$V_P = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \qquad \text{and} \qquad V_S = \sqrt{\frac{\mu}{\rho}}$$

It is clear from these expressions that the P-wave velocity is always larger than the S-wave velocity and that the S-wave is not viable in a medium with no shear strength.

Why the shear modulus is involved in the P-wave velocity is not intuitively obvious. The above derivation for the wave motion in a fluid is based on the fact that the bulk modulus K relates the volumetric strain to a *hydrostatic* pressure field. Although not depicted in the cross section the volume element is deformed equally in each coordinate direction – the moving pressure field does not change the *shape* of the element. In a solid the pressure disturbance is not hydrostatic and the shape of the element changes under deformation. It is the difference in deformation in different directions that brings the shear modulus into play in the resulting equations of motion.

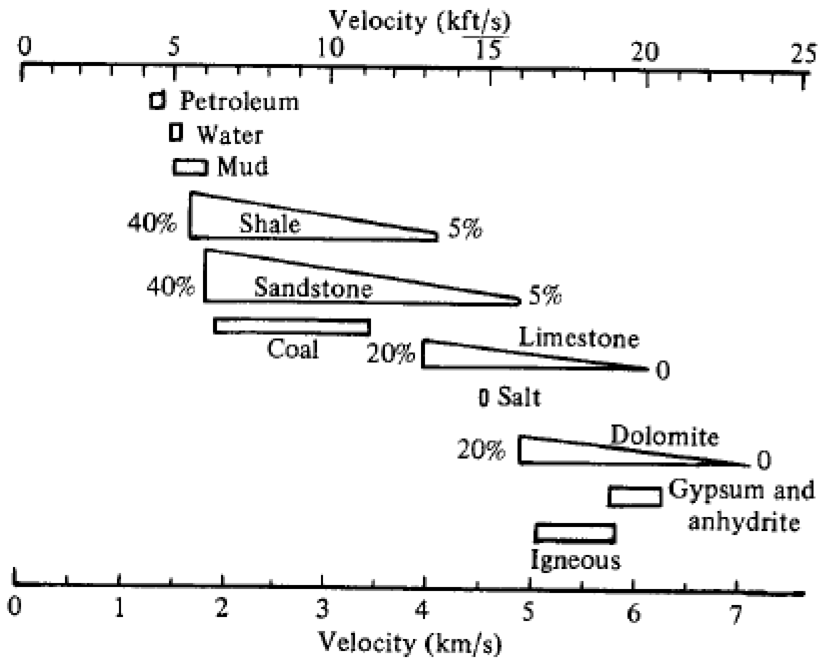
The particle velocity in a shear wave can be in any direction in the plane perpendicular to the propagation direction and it has become conventional to describe a shear wave by a component whose particle movement is in the horizontal direction, the S_H wave and a component whose particle motion is in a vertical plane, the S_V wave. Any S-wave propagating in any direction can always be represented by a superposition of S_H and S_V waves.

The elastic constants are all interrelated and a consequence that is very useful in the study of soils and rocks is that Poisson's ratio (ν) can be determined from the P and S wave velocities via the following equation;

$$\frac{V_P}{V_S} = \sqrt{\frac{2(1-\nu)}{1-2\nu}} \qquad \text{and so,} \quad \nu = \frac{V_P^2 - 2V_S^2}{2\left(V_P^2 - V_S^2\right)}$$

Poisson's ratio is theoretically bounded between 0 and 0.5 and for most rocks lies around 0.25, so typically V_P/V_S is about 1.7.

The values of P wave velocities in representative crustal rocks are shown in the following Figure:



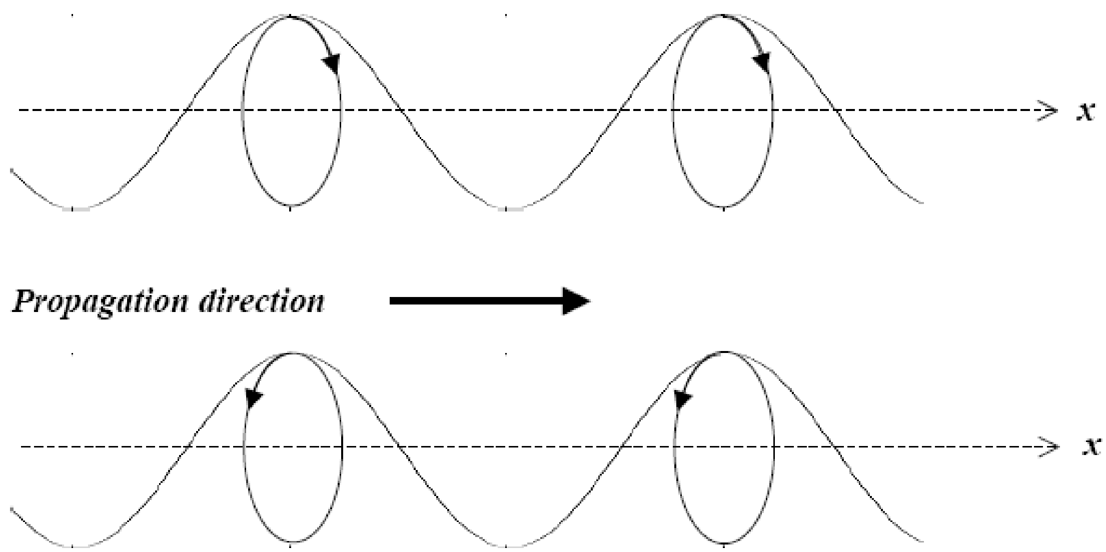
4. Surface waves

On the surface between a solid or liquid and air the deformation of an element is unconstrained in the direction normal to the surface. This changes the equation of motion for elements or particles adjacent to the surface and gives rise to another wave type commonly called the surface wave. Water waves are familiar and are readily seen as having a much lower velocity than a sound wave propagating through the bulk of the medium. Further a particle moves up and down vertically as well as in the direction of the wave. In detail a particle traces an ellipse with a *prograde* rotation as shown in the sketch below.

The surface wave on an isotropic half -space is known as a Rayleigh wave and it is similar in form to the surface wave on a liquid half-space except that the particle motion is *retrograde*. Rayleigh waves carry a large

amount of energy away from surface sources and are the noticeable, or ‘felt’, ground motion when one is standing near a seismic source. In exploration seismology Rayleigh wave are known as ground roll.

Surface wave on fluid – prograde particle motion

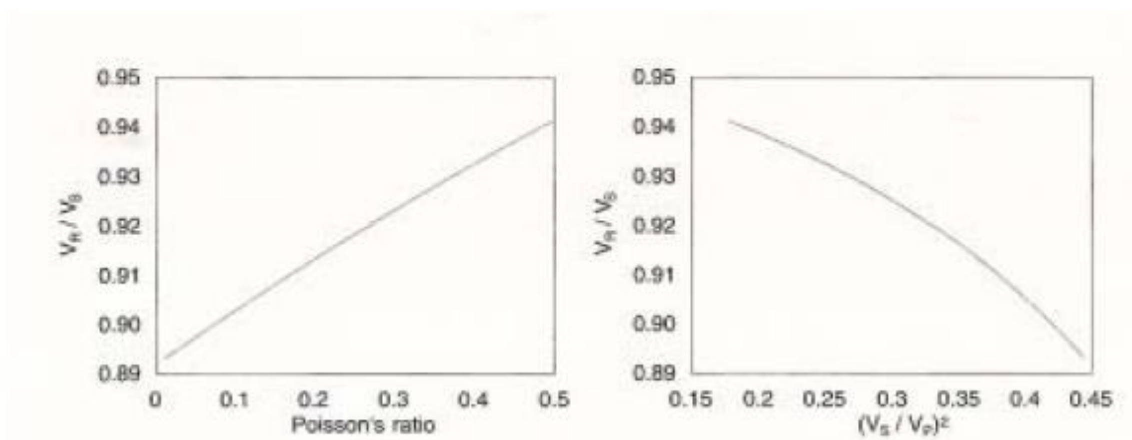


Surface wave on solid – retrograde particle motion

The velocity of a Rayleigh wave, V_R, is tied to the S-wave velocity and Poisson's ratio (and hence to V_P) through the solution to the following equation (White, 1983)

$$\left(2-\left(V_R/V_S\right)^2\right)^2 - 4\left(1-\left(V_R/V_P\right)^2\right)^{\frac{1}{2}}\left(1-\left(V_R/V_S\right)^2\right)^{\frac{1}{2}} = 0$$

The two plots below are taken from Mavko et al., 1998.



They show that for typical values of Poisson's ratio the Rayleigh wave velocity varies only from 0.91 to 0.93 V_S.

The amplitude of the particle displacement for surface waves decreases exponentially beneath the surface and the exponent is proportional to the frequency as well as the elastic constants and density of the medium. Since it is observed that velocity generally increases with depth, Rayleigh waves of high frequency penetrate to shallow depth and have *low* velocity whereas Rayleigh waves of low frequency penetrate to greater depth and have *high* velocity. The change in velocity with frequency is known as *dispersion.* For a typical surface source of Rayleigh waves the initiating disturbance has a broad frequency spectrum so the observed surface motion at some distance from the source is spread out in time, with low frequencies coming first and high frequencies coming later.

Measured Rayleigh wave dispersion can be used to invert for the vertical velocity profile.

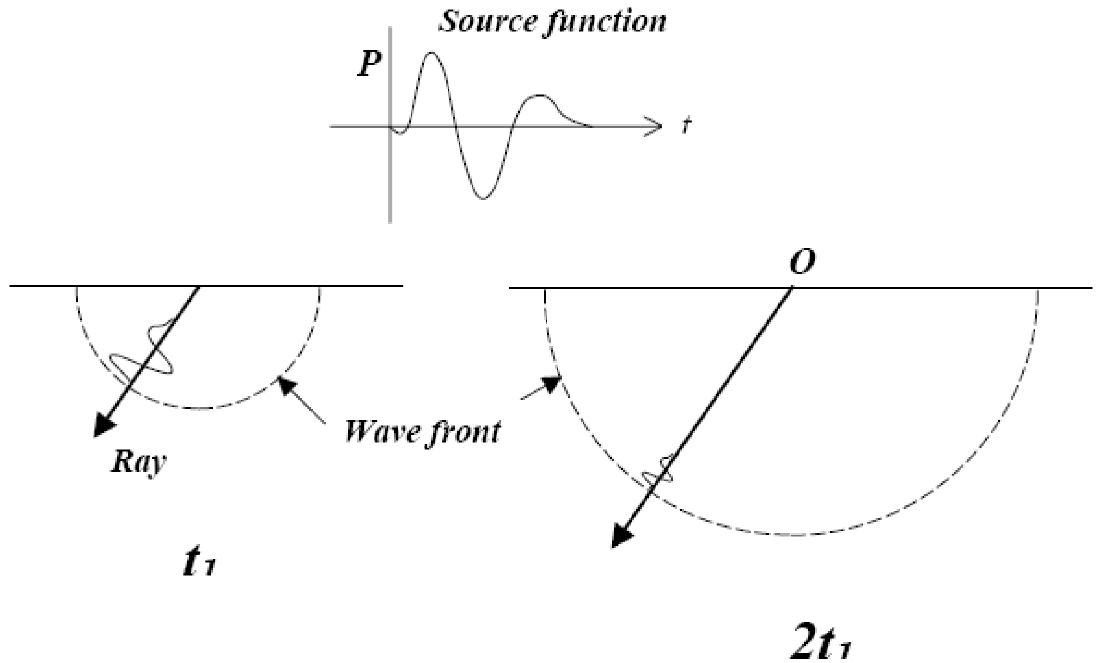
Finally, where the shear wave velocity increases with depth from the surface, Love showed that a horizontally polarized surface wave could exist. This wave, now called a Love wave, is also dispersive, and measurements of

it's dispersion can also be inverted to find the distribution of shear wave velocity with depth. Love waves require a distinct shear component in the source stress field so they are not a strong component of the disturbance propagated from the explosive or surface impact sources used in seismic exploration. They can be, and are, generated by special horizontal shear sources. For many shallow earthquakes they may be the dominant surface wave. This is a major factor used in discriminating between underground nuclear explosions and shallow earthquakes.

5. Wavefronts, rays and geometrical attenuation

Until now we have concentrated on the physics of stress-strain fields and their propagation in homogeneous solids and liquids. The fields arising from actual natural or artificial sources and what happens to such waves when they encounter inhomogeneities in density or elastic modulii is the real subject of interest. These are the properties that we wish to extract from measurements of the waves that have propagated into the earth.

To begin, an assumption is made that a point source is responsible for the resulting seismic wave. In the case of an explosive source it is assumed that an impulse of radially directed compressive force is created at a point. Similarly a hammer hitting a small steel plate on the surface is considered a point source of vertical compressive force. Such simple concepts immediately run into practical limitations (what if the ground surrounding the explosive is itself inhomogeneous, or the charge is close to the free surface? The stress field of the steel plate is not hemispherical and there is, in practice a strong component of shear stress along the edges of the plate.).



Considering the ideal compressive source at point O on a homogeneous half-space, the resulting seismic P-wave propagates such that all the points of constant phase lie on a hemisphere centered at O. The surface of all points of the same phase is called a wave front although the inferred meaning is that the phase in question is associated with some identifiable first arrival of the wave. A more rigorous definition is that the wave front is the surface of all equal travel times from the source. A cross section of such a hemispherical spreading wave is shown in the following cartoon for two successive time steps, t_1 and $2t_1$.

The wave in this cartoon is really a *wavelet*, the impulsive short wave train generated by the first compression of the source region, its subsequent recovery and perhaps a few highly damped oscillations. (The source deformation is large, not infinitesimal, and non-linear. Energy is dissipated in irreversible deformation (damage) to the surroundings, and the pulse dies quickly as suggested by the cartoon 'source function').

For convenience is describing wave propagation, the vector perpendicular to a wave front is defined as a *ray*. The ray in the cartoon is directed along a radius from the source, but this is not always the case. Rays are useful for describing what happens to waves when they pass through an interface and we will use them later to describe reflecting and refracting waves in layered media.

A numerical simulation of a seismic P-wave propagating away from a point source is shown in the following movie. In the movie the amplitude of the wavelet is contoured and color-coded. The high amplitude values are in red and the low amplitude in blue in this plot. The zero of the wavelet is the color of the undistorted background half-space. The time traces of geophones located in a well in the right hand side of the section are shown in the graph on the right of the figure. This is the fundamental form of the traces recorded in the vertical seismic profiling (VSP) method.

6. Amplitude and energy

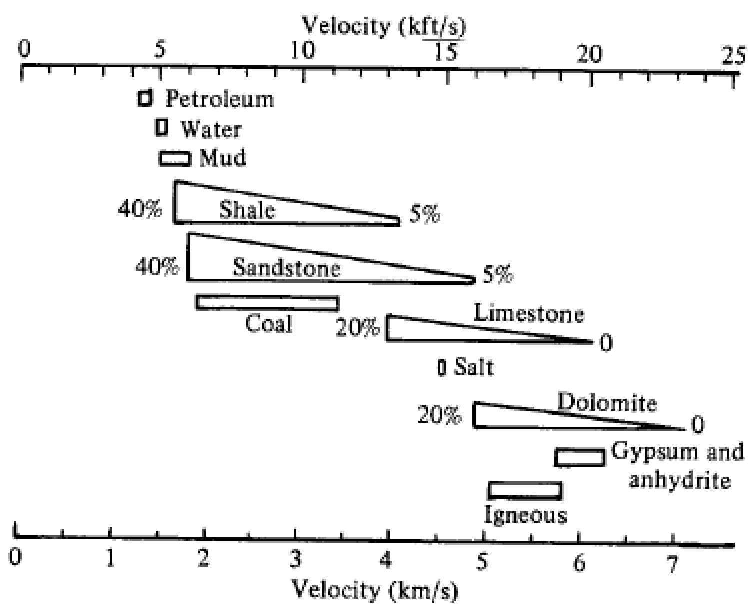
The energy imparted to the wave is proportional to the square of the amplitude of the resulting propagated wavelet. Barring actual non-elastic losses in the medium, the energy is conserved but is spread over the hemispherical shell that contains the wavelet. The energy in a volume element of this shell at radius r is roughly the total energy of the disturbance, E_0, divided by the area of the hemisphere, $2/3\pi r^2$. The energy in the wavelet thus falls off, or attenuates, as $1/r^2$. The energy in the wavelet is proportional to the square of the amplitude of the wavelet, so the amplitude of the spherically spreading wave falls off as $1/r$. The second wave front at $2t_1$ in the above cartoon is twice as far from the source as the first and so the amplitude of the wavelet is reduced by half. This is called the geometric attenuation and it is separate from any other attenuation due to loss mechanisms in the medium. Surface waves spread in circular patterns from a point source (think water waves spreading from the impact point of a stone) and in this case the energy decays as $1/r$ from the source and the amplitude consequently falls as $1/\sqrt{r}$.

At very large distances from the source, the curvature of the wave front becomes small and in some situations it is possible to neglect the curvature and assume that the wave front is a plane. Such a wave is then known as a *plane wave*.

Seismic velocity, attenuation and rock properties

- **Rock properties that affect seismic velocity**
  - Porosity
  - Lithification
  - Pressure
  - Fluid saturation
- **Velocity in unconsolidated near surface soils (the weathered layer)**
- **Attenuation**

Seismic surveys yield maps of the distribution of seismic velocities, interfaces between rock units and, ideally, of reflection coefficients at these interfaces. The velocities of crustal rocks vary widely as the following figure shows.



Generally, the velocities depend on the elastic modulii and density via:

$$V_P = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \qquad \text{and} \qquad V_S = \sqrt{\frac{\mu}{\rho}}$$

These elastic constants, and densities, in turn depend on the properties that the geologist or engineer use to characterize the rock such as porosity, fluid saturation, texture etc. A review of the relationships between the intrinsic rock properties and the measured velocities or reflectivities is needed before seismic survey results can be interpreted quantitatively in terms of lithology. Many of these relationships are empirical – velocities are found to be related to certain rock units in a given locale by actual laboratory measurements on core samples of the rock or soil.

It is observed from seismic surveys that velocities generally increase with depth. Densities also increase with depth so it must be that the bulk and shear modulii increase faster than the density. In seismic exploration there are many empirical relationships between velocity and depth of burial and geologic age.

The relationship between intrinsic rock properties such as porosity, fracture content, fluid content and density and velocity underlie the empirical relationships mentioned above.

Rock properties that affect seismic velocity

1) Porosity. A very rough rule due to Wyllie is the so called time average relationship:

$$\frac{1}{V_{bulk}} = \frac{\phi}{V_{fluid}} + \frac{1-\phi}{V_{matrix}}$$

where φ is the porosity.

This is not based on any convincing theory but is roughly right when the effective pressure is high and the rock is fully saturated.

2) Lithification.

Also known as cementation. The degree to which grains in a sedimentary rock are cemented together by post depositional, usually chemical, processes, has a strong effect on the modulii. By filling pore space with minerals of higher density than the fluid it replaces the bulk density is also increased. The combination of porosity reduction and lithification causes the observed increase of velocity with depth of burial and age.

3) Pressure.

Compressional wave velocity is strongly dependant on effective stress. [For a rock buried in the earth the confining pressure is the pressure of the overlying rock column, the pore water pressure may be the hydrostatic pressure if there is connected porosity to the surface or it may be greater or less than hydrostatic. The effective pressure is the difference between the confining and pore pressure.]

In general velocity rises with increasing confining pressure and then levels off to a “terminal velocity” when the effective pressure is high. The effect is probably due to crack closure. At low effective pressure cracks are open and easily closed with an increase in stress (large strain for low increase in stress—small K and low velocity). As the effective pressure increases the cracks are all closed, K goes up and the velocity increases.

Finally even at depth, as the pore pressure increases above hydrostatic, the effective pressure decreases as does the velocity. Over pressured zones can be detected in a sedimentary sequence by their anomalously low velocities.

4) Fluid saturation.

From theoretical and empirical studies it is found that the compressional wave velocity decreases with decreasing fluid saturation. As the fraction of gas in the pores increases, K and hence velocity decreases. Less intuitive is the fact that V_s also decreases with an increase in gas content. The reflection coefficient is strongly affected if one of the contacting media is gas saturated because the impedance is lowered by both the density and velocity decreases.

Velocity in unconsolidated near surface soils (the weathered layer)

The effects of high porosity, less than 100% water saturation, lack of cementation, low effective pressure and the low bulk modulus (due to the ease with which native minerals can be rearranged under stress) combine to yield very low compressional and shear wave velocities in the weathered layer. V_p can be as low as 200 m/sec in the unsaturated zone (vadose zone) – less that the velocity of sound in air!

Attenuation

It is observed that seismic waves decrease in amplitude due to spherical spreading and due to mechanical or other loss mechanisms in the rock units that the wave passes through.

The attenuation for a sinusoidal propagating wave is defined formally as the energy loss per cycle (wave length) $\Delta E/E$ where E is the energy content of the wave.

Mathematically, the propagating wave $A = A_0 e^{i\omega t - ikx}$, get an added damping term $e^{-\alpha x}$, so the solution becomes $A = A_0 e^{i\omega t - ikx} e^{-\alpha x}$

[We can apply this to the definition of attenuation $\Delta E/E$ by substituting A^2 for the energy at two points at distance λ (the wavelength) apart and we find $\frac{\Delta E}{E} = 2\alpha\lambda$

There are many theories for explaining attenuation in rocks. Friction, included by including a velocity term in the governing differential equation for the displacement does <u>not</u> explain laboratory measurement. Various other damping mechanisms such as viscous flow (Biot Theory) have some success but much important work remains to be done in this area (especially for unconsolidated material where the attenuation is very high). Some of the theories predict attenuation as well as dispersion (the variation of velocity with frequency).

Experimentally it is found that the attenuation coefficient α depends on frequency and that there is little dispersion. In fact to a good approximation attenuation can be described by $A = A_0 e^{-\beta f x}$. With x in meters and f in Hertz, a typical shale has a $\beta = 10^{-4}$. So at one Hertz the amplitude falls to A_0/e at 10 km. But at 1000 Hz it falls to A_0/e in 10 m. The attenuation may be as much as 10 times greater in unconsolidated sediments.

Another important attenuation mechanism is the reduction in amplitude of a wave by the scattering of its energy by diffraction by objects whose dimensions are on the order of the wavelength. If a is an average linear dimension of velocity inhomogeneities then the attenuation coefficient is given approximately by:

$$\alpha \approx \frac{a^3}{\lambda^4}$$

So attenuation increases rapidly with decreasing wavelength. Consider attenuation is an unconsolidated medium with a velocity of 250 m/sec and a frequency of 1000 Hz. Then, $\lambda = 0.25$ m, and $\alpha = a^3 \times 256$. The wave would fall to 1/e of its initial amplitude when a = 157 m.

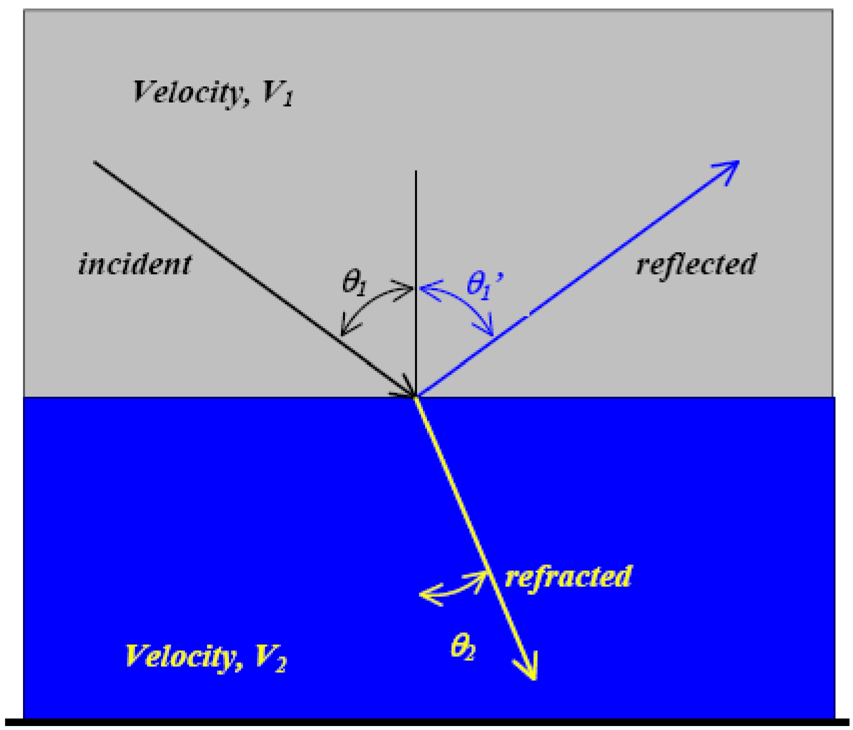
It might be reasonable to expect inhomogeneities with a characteristic dimension on the order of 15 cm in the overburden so it is likely that the very high attenuation observed in near surface unconsolidated sediments is due to scattering.

Reflection and Refraction

1. The geometry of reflection and refraction

A wave incident on a boundary separating two media is reflected back into the first medium and some of the energy is transmitted, or refracted, into the second. The geometry of refraction and reflection is governed by Snell's Law which relates the angles of incidence, reflection and refraction to the velocities of the medium.

The cartoon below illustrates the ray geometry for a P-wave incident on the boundary between media of velocity V_1 and V_2. The angles of incidence, reflection and refraction, θ_1,θ_1', and θ_2, respectively are the angles the ray makes with the normal to the interface.



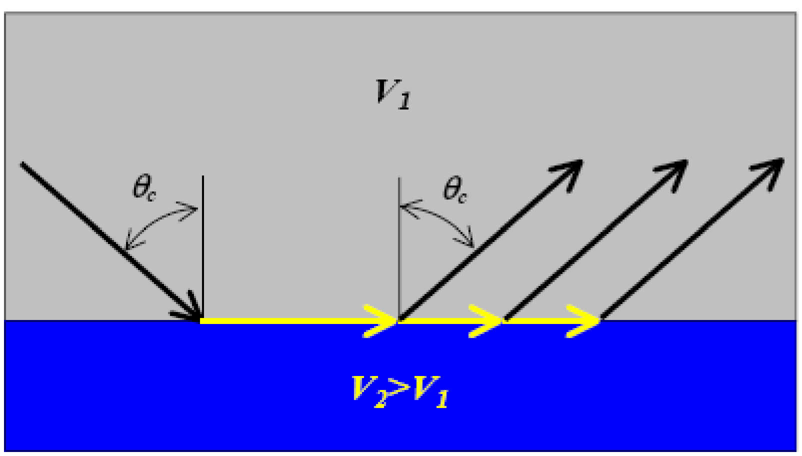
Snell's Law states that: $$\frac{Sin\,\theta_1}{V_1} = \frac{Sin\,\theta_1'}{V_1} = \frac{Sin\,\theta_2}{V_2}$$

Snell's law requires that the angle of reflection is equal to the angle of incidence. It further implies that if $\boldsymbol{V_2}$ is less than $\boldsymbol{V_1}$ the ray is bent towards the normal to the interface as in the cartoon, but if $\boldsymbol{V_2}$ is greater than $\boldsymbol{V_1}$ the ray is bent away from the normal.

It is instructive to observe the progress of a spherically spreading wave as it impinges on a horizontal boundary between media of different velocity. The wave fronts are definitely not planar nevertheless rays at the interface satisfy Snell's Law. As in the planar model used to derive Snell's Law, each point on the interface where the spreading wave hits becomes the source of new waves which propagate back into medium one and on into medium 2. In the following movie the spreading wave is reflected at the interface and refracted into a medium of lower velocity.

If V_2 is greater than V_1 the angle of refraction is greater than the angle of incidence. The latter result can lead to a special condition where $\theta_2 = \mathbf{90°}$. The angle of incidence for which this occurs is called the *critical angle*, θ_c. The critical angle is given by; $\theta_c = Sin^{-1}\left(\frac{V_1}{V_2}\right)$

The geometry of this wave is correctly given by Snell's law but the nature of a wave that propagates along the interface is not so simple. The critically refracted wave is a disturbance that propagates along the interface with the velocity of the lower medium. As it goes its wave front acts as a moving source of waves that propagate back into the upper medium with velocity V_1 along rays which are parallel to the reflected wave at the critical angle. This phenomenon is illustrated in the following cartoon.



The following movie shows the propagation of an expanding wav front as it moves through such a boundary.

The wave front in the lower, higher velocity, layer moves faster than the incident wave front along the boundary. In the plane of the boundary this creates a stress disturbance which travels along the boundary with velocity V_2 and at each point acts as a source for waves propagating back up to the surface. This wave is commonly known as a Head wave and its wave front is clearly seen beyond the point on the interface where the incident ray is at the critical angle.

In the description of reflection and refraction up till now we have not discussed the physics of why a seismic wave is reflected only what the geometric relationship of the wave fronts must be as the wave crosses an interface. The energy that is reflected is determined by using the form for the solution for the particle displacement (e.g. the form $A_0 e^{i(\omega t - kx)}$ that we saw for a wave traveling in the x direction) for a wave traveling towards an interface (the incident wave), one traveling away from the interface in the opposite direction to the incident wave (the reflected wave) and finally one transmitted into the second medium. At the interface these general solutions must satisfy the boundary conditions: the displacement across the interface must be continuous and both the normal and tangential stress must be continuous.

For a P wave incident at an arbitrary angle these boundary conditions cannot be met without incorporating an S_V wave as a reflected and transmitted wave. Thus it appears that the incident P wave generates an S_V wave at the interface. Note that the S_V waves leave the interface at angles dictated by <u>Snell's Law</u>:

$$\frac{\sin\theta_{1P}}{V_{1P}} = \frac{\sin\theta_{2P}}{V_{2P}} = \frac{\sin\theta_{1S}}{V_{1S}} = \frac{\sin\theta_{1S}}{V_{2S}}$$

The solutions for the amplitudes of the various P and S_V waves for a P wave incident at an arbitrary angle are given by the Zoeppritz Equations. In general they yield complex results that cannot be summarized in this introductory treatment. The analysis of the amplitude of the reflected waves as a function of incident angle, which in practice means as a function of shot-point geophone off-set, yields valuable information about the rock properties on either side of the interface. This forms the basis of the important amplitude vs. offset (AVO) analysis of modern reflection seismology.

The reflection coefficient for small off-sets is very close to that for normal incidence, and since for normal incidence the stresses tangent to the interface are zero, the dependence on the shear component disappears and no shear wave is generated. The reflection coefficient from the Zoeppritz equations takes on a very simple form:

$$\frac{Amplitude\ of\ reflected\ wave}{Amplitude\ of\ incident\ wave} = \boldsymbol{R} = \frac{\rho_2 V_2 - \rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1}$$

Similarly the transmission coefficient is:

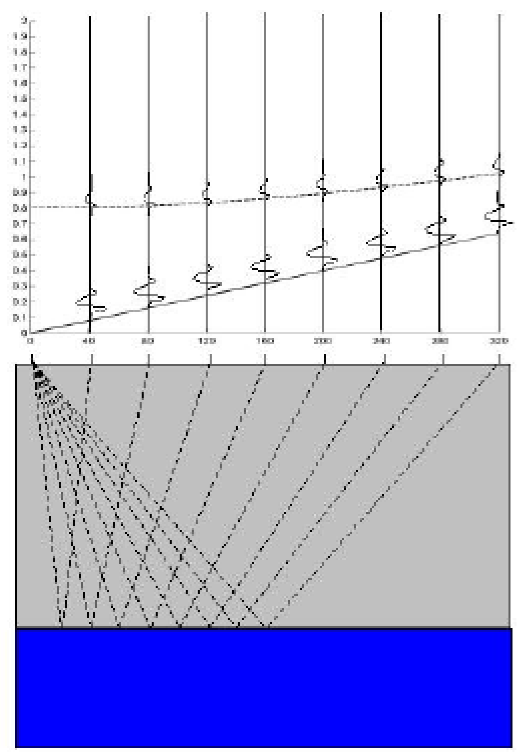
$$\frac{Amplitude\ of\ transmitted\ wave}{Amplitude\ of\ incident\ wave} = \boldsymbol{T} = \frac{2\rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1}$$

Note that it is the energy that is conserved in reflection and transmission and the energy is proportional to the square of the amplitudes of the waves. The energy reflection coefficients are the squares of the above amplitude reflection coefficients. This is important because it will be noted that the sum of the fraction of the amplitude reflected and the fraction transmitted do not generally add up to one whereas the energy coefficients do.

Time – Distance Plots

- Reflection time-distance plots
- Moveout
- Dip moveout
- Reflection survey configuration
- Geophone arrays and spacial filtering
- Migration
- Refraction time-distance plots
- The ray-tracing algorithm

In surface seismic surveys the 'point' source is located on the surface and detectors of the resulting seismic waves are located on the surface. The data of a survey are the arrival times of the wave fronts at various distances from the source. We have already seen a sample of this in the seismic time-traces that would be observed at geophones placed in a well adjacent to a surface source. The data are usually plotted with the arrival time on the vertical axis and the separation on the horizontal axis. The following cartoon shows a hypothetical surface reflection survey in which an array of 8 geophones is placed along a line on the surface at equal intervals from the source, S (usually called the shot point). The geophones in this example are located at 40, 80, 120, to 320 m, the layer is 200 m thick and the velocity is 500m/s.



The plot of wavelet arrivals vs. time at any particular geophone location is a recording of the ground motion at that geophone. This is the data recorded in a seismic survey and it is usually called the seismogram. The above schematic result is consequently known as a synthetic seismogram. The actual seismogram is considerably more complex because it displays the ground roll, refractions if there are any, and shear wave arrivals from part of the incident wave energy that is converted to shear energy at the interface.

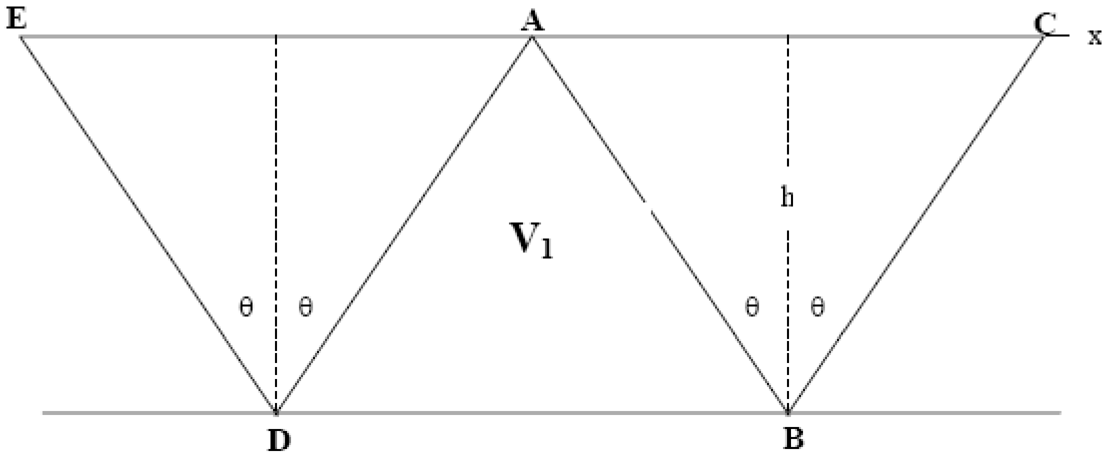
The travel time curves for models with layered dipping interfaces can be calculated analytically and these formulas are well described in the standard texts referred to in the introduction to this chapter. These solutions should always be used to check any of the more general modeling codes.

The real world is rarely uniformly layered, certainly not with uniform layers of constant velocity. It is known that in sedimentary rocks the velocity increases with depth even in what appears to be a uniform depositional sequence. Further, velocities can vary laterally in a given geological unit because of depositional variation in grain size, clay content or degree of cementation. Finally the subsurface has structure. The goal of shallow surveys is often to map the depth to bedrock and this bedrock interface is unlikely to be a planar surface. Sedimentary layers have faults, anticlines, folds, and unconformities which are in fact the very features that trap petroleum and are the targets of the seismic exploration program in the first place.

The major task of modern exploration seismology is to develop models of the subsurface and methods of data processing which can be used to interpret the complex wave front arrivals on a typical seismogram. The numerical modeling programs that are used to create synthetic seismograms range from full 3 dimensional (3D) finite element or finite difference solutions to the governing wave equation to approximate solutions that trace the progress of particular rays through the medium. In this course we have adopted a general ray tracing program for creating travel time curves. This code will be used for modeling reflections and refractions from simple planar interfaces in the discussion that follows.

Reflection time- distance plots

Consider a source (shot point) at point A with geophones spread out along the x-axis on either side of the shot point.



A raypath from A to C or A to E is: $2\sqrt{h^2 + \left(\frac{x}{2}\right)^2}$

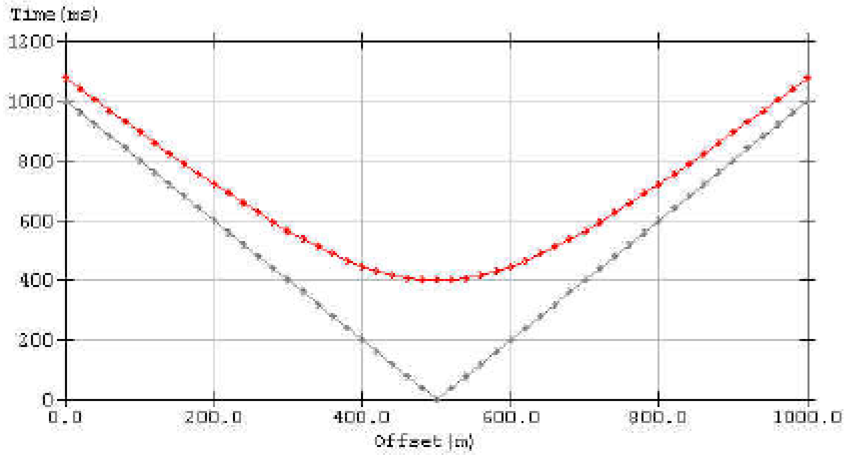
The travel time, t, is the raypath divided by the velocity, V_1, or:

$$t = \frac{\sqrt{x^2 + 4h^2}}{V_1}$$

Rearranging:

$$\frac{V_1^2 t^2}{4h^2} - \frac{x^2}{4h^2} = 1$$

This is the equation of a hyperbola symmetric about the t axis. The travel time plot for the direct wave arrivals and the reflected arrivals are shown in the following plot. The first layer is 100 m thick and its velocity is 500 m/s. The intercept of the reflected arrival on the t axis, t_i, is the two-way zero offset time and for this model is equal to 400 ms. At large offsets the hyperbola asymptotes to the direct wave with slope $1/V_1$.



In most seismic reflection surveys the geophones are placed at offsets small compared to the depth of the reflector. Under this condition an approximate expression can be derived via:

$$t^2 = \frac{4h^2}{V_1^2} + \frac{x^2}{V_1^2}$$

which can be rewritten as;

$$t = \frac{2h}{V_1}\left[1 + \left(\frac{x}{2h}\right)^2\right]^{\frac{1}{2}}$$

or since $\frac{2h}{V_1} = t_i$,

$$t = t_i\left[1 + \left(\frac{x}{V_1 t_i}\right)^2\right]^{\frac{1}{2}}$$

Since $\frac{x}{V_1 t_i}$ is less than 1, the square root can be expanded with the binomial expansion. Keeping only the first term in the expansion the following expression for the travel time is obtained:

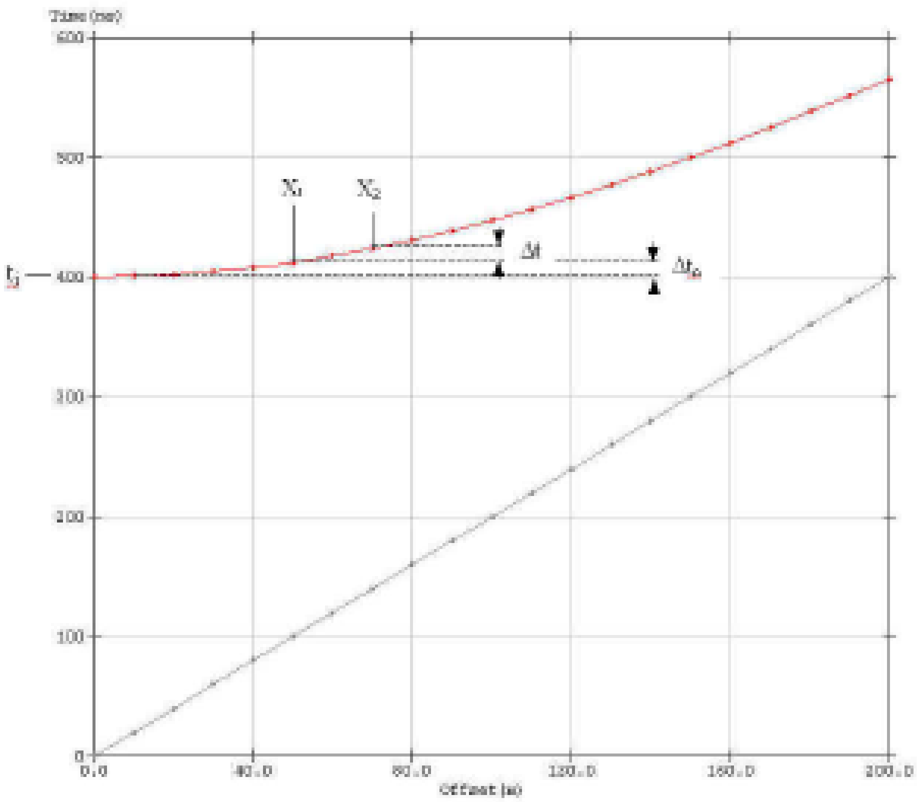
$$t = t_i\left[1 + \frac{1}{2}\left(\frac{x}{V_1 t_i}\right)^2\right]$$

This is the basic travel time equation that is used as the starting point for the

interpretation of most reflection surveys.

Moveout

A useful parameter for characterizing and interpreting reflection arrivals is the moveout, the difference in travel times to two offset distances. The following expanded plot of one side of the hyperbola of the previous reflection plot shows the moveout, Δt, for two small offsets.



Using the small offset travel time expression for x_1 and x_2 yields the following expression for the moveout:

$$\Delta t = \frac{x_2^2 - x_1^2}{2V_1^2 t_i}$$

The normal moveout (NMO), Δt_n, is a special term used for the moveout when x_1 is zero. The NMO for an offset x is then:

$$NMO, \Delta t_n, = \frac{x^2}{2V_1^2 t_i}$$

The NMO is readily measured with small offset reflection data. With the value of the intercept time, t_i, the velocity is determined via:

$$V_1 = \frac{x}{(2t_i \Delta t_n)^{1/2}}$$

and the depth is then determined by:

$$h = \frac{V_1 t_i}{2}$$

=

For a given offset the NMO decreases as the reflector depth increases and/or as the velocity increases.

In a layered medium the velocity obtained from the NMO of a deep reflector is an average of the intervening layer velocities. Dix (1955) found that the root-mean-square velocity defined by:

$$V_{rms} = \left(\frac{\sum_1^n V_i^2 t_i}{\sum_1^n t_i} \right)^{\frac{1}{2}}$$

where V_i is the velocity in layer i and t_i is the travel time in layer i is the best average to use.

In interpretation the NMO's for successive reflections are used to obtain the average velocity to each reflector. Assuming these are the V_{rms} velocities defined above then Dix (1955) showed that the velocity in the layer bounded by the n^{th} and $n-1^{th}$ layer is given by:

$$V_n = \left(\frac{V_{rms_n}^2 t_n - V_{rms_{n-1}}^2 t_{n-1}}{t_n - t_{n-1}} \right)^{\frac{1}{2}}$$

Dip moveout

If the interface is dipping as in the figure below the up-dip and down-dip travel times are changed by an amount dependant on the dip angle θ. The time-distance plot is still a hyperbola but the axis of symmetry is shifted up-dip by 2h sinθ. (Shown by the dashed line in the figure. Note also that the depth is still the perpendicular distance from the interface to the shot point). The binomial expansion for the travel time for small offsets becomes:

$$t = t_i \left(1 + \frac{x^2 + 4xh\sin\theta}{2V^2 t_i^2} \right)$$

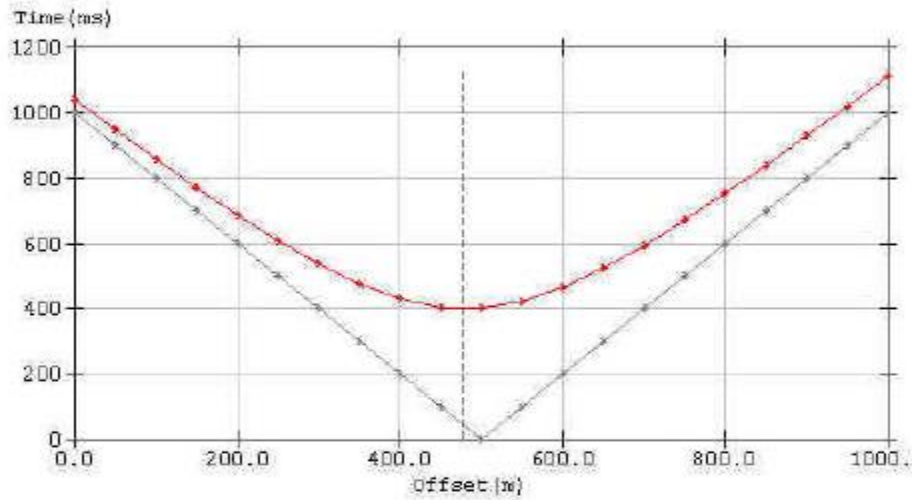
For geophones offset a distance x up-dip and down-dip, the dip moveout is defined as:

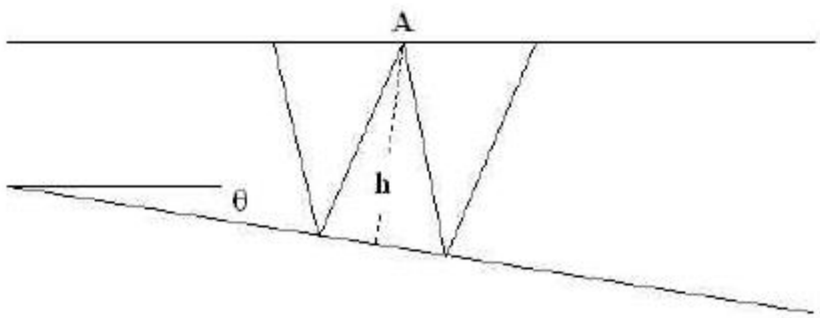
$$dip\ moveout = \Delta t_d = t_{+x} - t_{-x} = \frac{2x\sin\theta}{V}$$

For small dips when $\theta \approx \theta$ the dip moveout yields the dip via;

$$\theta \approx \frac{V\Delta t_d}{2x}$$

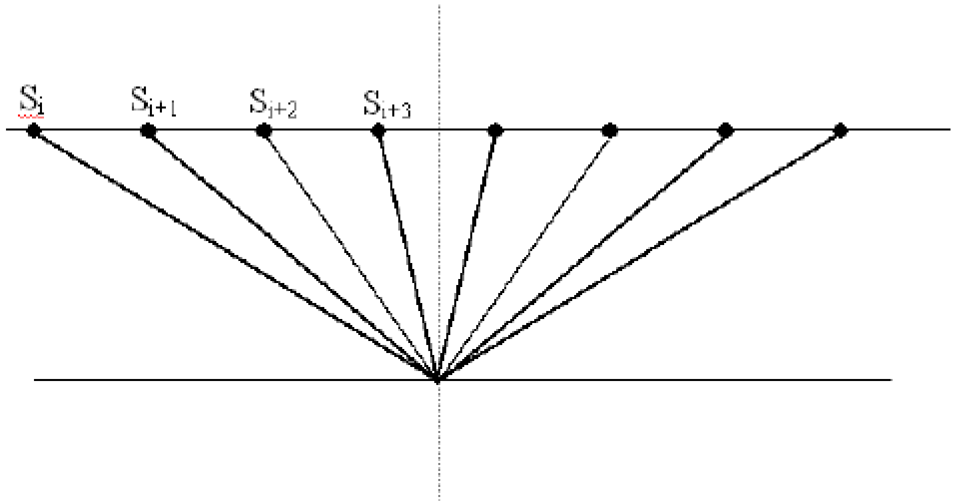
The velocity can be obtained with sufficient accuracy by averaging the velocities obtained in the usual manner from the up-dip and down-dip NMO's.





Reflection survey configuration

There have been many configurations of shot point and geophone arrays used over the years. One important array is illustrated in the figure below. The geophone array, also called the spread, is laid out almost continuously along the profile. Shots are placed at the same locations as the geophones. At each shot point, S_i, recordings are made of the seismic record at each n geophones on either side of S_i. After a succession of shots e.g. S_{i+1}, $S_{i+2,}$ $S_{i+3,}$ the geophone traces corresponding to rays that reflect at a common depth point (CDP) are collected and plotted.



This collection of records is also called a common mid point array. The advantage of such an array is that many reflections from the same portion of the reflecting interface can be averaged.

The resulting gather of traces will of course show the typical moveout of the reflector but now all the rays reflect off the same point (the assumption is made that the layer has a very small dip otherwise the rays will not have a common reflection point). In practice there are variations in moveout caused by near surface variations in velocity so the moveouts of each trace will vary but because the reflection point is common and an average moveout can be calculated from which the velocity can be obtained.

The background ground motions at separate geophones are assumed to be random, as are the variations in near surface velocity. With these assumptions one method of averaging is to assume a velocity and shift each trace back to its zero offset value by its moveout. If the data were perfect all the reflection arrivals would line up horizontally and the traces could all be added together to form an average zero offset trace. By successively

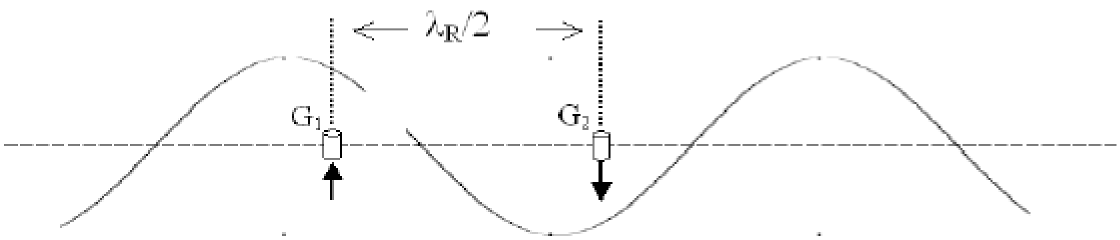
changing the velocity until the maximum average reflection is obtained the optimum velocity is determined. In this average trace the reflection event would be well defined but the adjacent noise would average towards zero. Even with reflections with variable moveouts, the average will lead to something greater than the noise average so this process still leads to the selection of an optimum velocity. This process of shifting and averaging the pairs of traces is called a CDP gather.

The final averaged time trace is plotted directly beneath mid point of the pairs making up the gather. In practice, up to 64 common mid point shot-receiver pairs may be averaged with this single CDP trace. (The number of pairs averaged in this

manner is referred to as the fold of the CDP gather.) The entire process is repeated to produce another CDP trace, one interval, Δ_x , farther along the spread.
For most reflection surveys the traces shown are CDP gathers.

Geophone arrays and spacial filtering

A major problem in reflection surveying is the presence of a large amplitude Rayleigh wave. Including the Rayleigh wave in a typical trace time plot for a deep reflection usually shows that the Rayleigh wave often arrives just in the time window of short offset reflections.
The Rayleigh wave can be minimized by considering the seismic arrivals at two geophones spaced at half the wavelength of the Rayleigh wave.



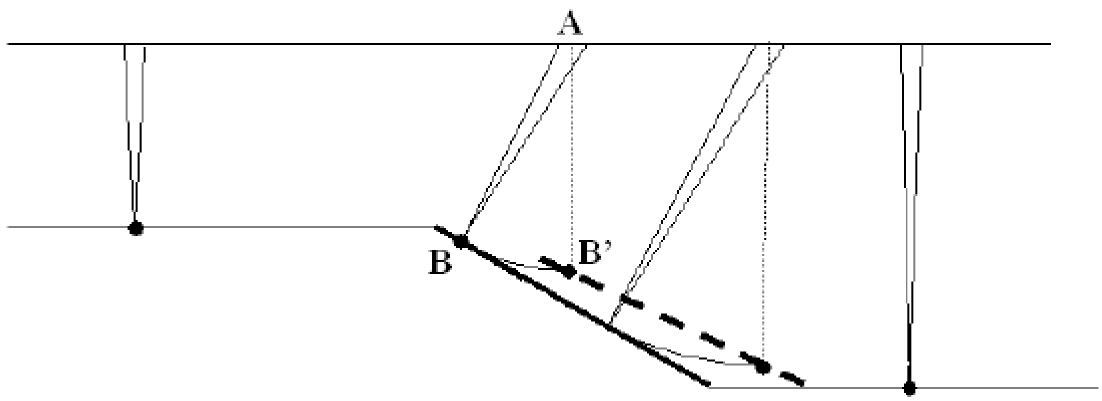
If the output of two such geophones is summed the Rayleigh wave will produce no output. The reflected wave on the other hand is coming up at near vertical incidence and will be doubled in the summed output. More geophones at the correct spacing will continue to augment the reflected arrival while effectively canceling the Rayleigh wave.
In practice each receiver location in a reflection survey consists of a group of geophones whose spacing is chosen to cancel the Rayleigh wave. A test survey is conducted first to determine the Rayleigh velocity and frequency from which the wave length is determined from $\lambda = V/f$.

Migration

For any reflection array the interpreted refection point is plotted directly beneath the mid point of the shot receiver separation. This is a plotting convention because information about the possible dip or reflector

geometry is not generally available in the simple offset data. The apparent vertical section is distorted by this means of plotting because the actual reflector point is plotted beneath the mid point.

Consider the following sloping step on a reflecting interface. Assume the velocity is known. The plots for the for the zero offset reflection (e.g. The CDP gather) are plotted on the model. On the left and right of the sloping section the reflection section mimics the actual section. However for reflections such as ABA, which occur from point B on the slope the plotted point is at B^{1-} displaced to the right of the actual reflection point. Its ‘depth’ is just where the arc of radius AB intersects the vertical beneath A. The net effect is that the plot of apparent reflection points shifts the interface to the right and changes its slope.

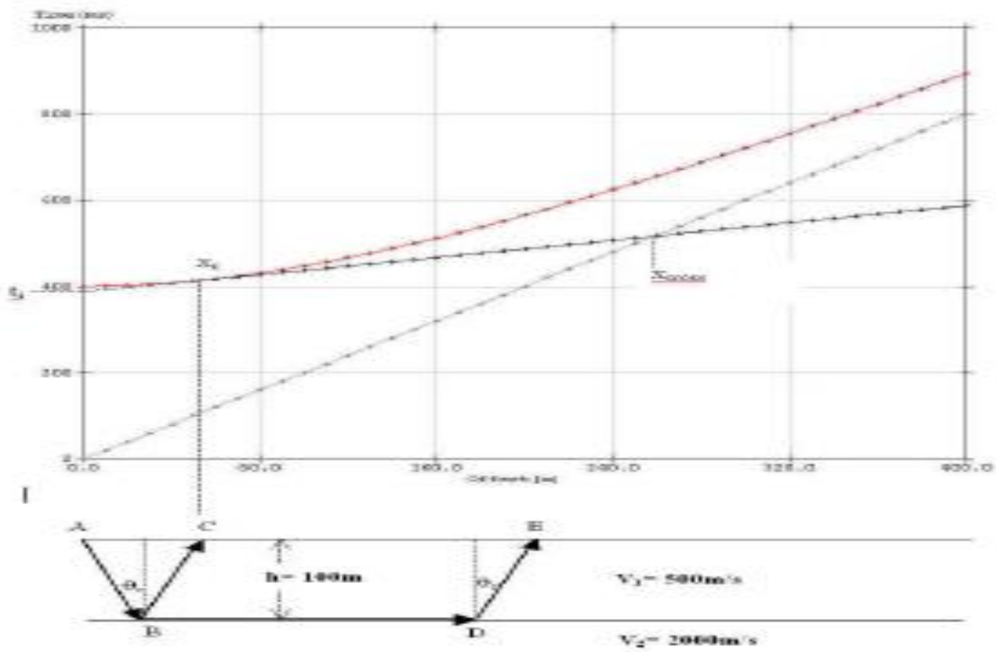


This description also suggests the means to correct the section. Whenever a sloping interface is found in the zero offset section the points on this interface are moved back along an arc centered at the shot point. The line tangent to all the arcs is the true position of the interface. The process

of shifting an apparent slope back to its true position in space is known as migration.

Refraction time-distance plots

A typical ray path for an incident ray refracted at the critical angle is made up of the lines ABDE shown in the figure below. The incident ray at the critical angle, AB, yields a reflection BC and generates the head wave which propagates along the interface. The wave front of the head wave generates waves which return to the surface along rays which leave the interface at the critical angle, e.g path DE in the figure. The refraction arrivals consequently begin at the same time as the reflected wave on path ABC. Subsequent refraction arrivals are delayed by their travel time along the interface at the velocity of the lower medium.



The equation for the travel time to an arbitrary point on the surface is the sum of the travel times along AB, BD, and DE. The first and third times are identical so:

$$t = t_{AB} + t_{BD} + t_{DE}(= t_{AB})$$

$$t = \frac{2AB}{V_1} + \frac{BD}{V_2}$$

Using the geometry imposed by Snell's Law this becomes:

$$t = \frac{2h}{V_1 \cos\theta_c} + \frac{x - 2h\tan\theta_c}{V_2}$$

Since θ_c is determined via the velocities, $\sin\theta_c = \frac{V_1}{V_2}$, then the equation can be

(note $\cos\theta_c = \frac{\sqrt{V_2^2 - V_1^2}}{V_1}$

rewritten in terms of velocity as:

$$\tan\theta_c = \frac{V_1}{\sqrt{V_2^2 - V_1^2}})$$

$$t = \frac{x}{V_2} + \frac{2h\sqrt{V_2^2 - V_1^2}}{V_1V_2}$$

This is the equation of a straight line with slope $1/V_2$ and an intercept on the t axis,

$$t_i = \frac{2h\sqrt{V_2^2 - V_1^2}}{V_1V_2}.$$

This is the mathematical intercept; there are no refracted arrivals at distances less than AC or at times less than the reflection travel time for the ABC path.

The velocities can be determined directly from the travel time plot as the inverse of the slopes of the direct and refracted arrivals so the depth can be determined from the intercept time via:

$$h = \frac{t_iV_1V_2}{2\sqrt{V_2^2 - V_1^2}}$$

The distance AC at which the first refraction arrives, called the critical distance, x_c , can be obtained from:

$$\frac{x_c}{2h} = \tan\theta_c = \frac{V_1}{\sqrt{V_2^2 - V_1^2}}$$

so

$$x_c = \frac{2hV_1}{\sqrt{V_2^2 - V_1^2}}$$

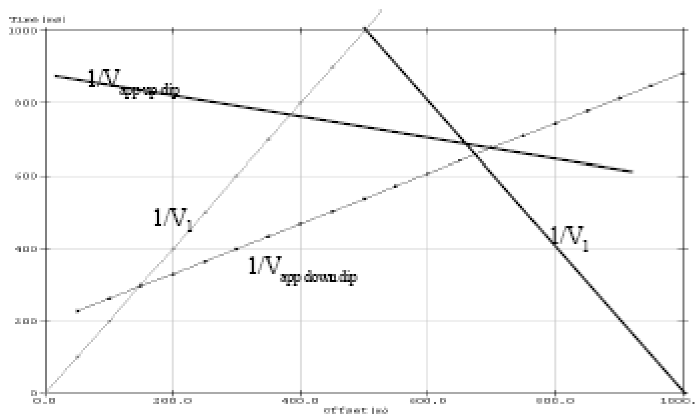
Finally it can be seen from the time-distance plot that there is a distance after which the refracted arrivals come before the direct arrivals. This occurs at the crossover distance, x_{cross}, when the refraction and direct waves have equal travel times, i.e when

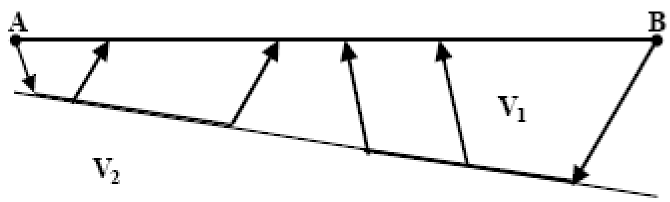
$$\frac{x_{cross}}{V_1} = \frac{x_{cross}}{V_2} + \frac{2h\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

or when
$$x_{cross} = 2h\left(\frac{V_2 + V_1}{V_2 - V_1}\right)^{\frac{1}{2}}$$

This is another useful equation for determining h. In practice with real data it is usually found that projecting the refracted arrivals back to the t axis to find the intercept time is more accurate than estimating where the crossover distance is.

The refraction arrivals from shot points at each end of a survey line over a dipping interface are shown in the following figure:





The arrivals at geophones down dip from shot point A come at progressively later times than their horizontal interface counterparts so that the slope of the arrival curve is steeper. The apparent velocity obtained from the plot, $V_{app\ down\ dip}$, is less than $V_{2.}$ The apparent up dip velocity obtained with geophones up dip from shot point B is greater than V_2. The travel times from A to B and from B to A, the reciprocal times, must be the same. Refraction surveys must be shot in both directions. Arrival times taken in only one direction and interpreted as being taken over a horizontal interface may yield erroneous results if the interface is dipping.

The equations for the travel times for a dipping interface, and for multiple layers with dipping or horizontal interfaces, are derived analytically in Telford et al.(1990) and they present a useful collection of expressions for finding the depths and dips for up to three layer models.

A particularly useful result for small dips is that

$$\frac{1}{V_2} \approx \frac{1}{2}\left(\frac{1}{V_d} + \frac{1}{V_u}\right)$$

where V_d and V_u are abbreviations for the down dip and up dip apparent velocities respectively.

General expressions have been derived for the travel times for any number of layers with accompanying equations for depths and true velocities but the quality of the field time-distance data makes it difficult to identify intercept times or cross over distances for more than a few refraction arrival segments. A better approach which leads into general methods of interpreting seismic data is to use a numerical technique to generate arrivals in model of an arbitrary medium and then by a process known as inversion adjust the parameters of the model to match the observed data.

In summary the principal advantage of the refraction method over the reflection method is that it depends only on measuring the first arrival times on a seismic time trace. There is no problem separating the refracted arrival from other arrivals as there is in picking reflection events. Problems or disadvantages are:

i) there is no evidence in the travel time plot for an intermediate layer(s) of lower velocity than the layers enclosing it. Interpretation in this case, which assumes a progressive increase in layer velocity with depth, will be in error.

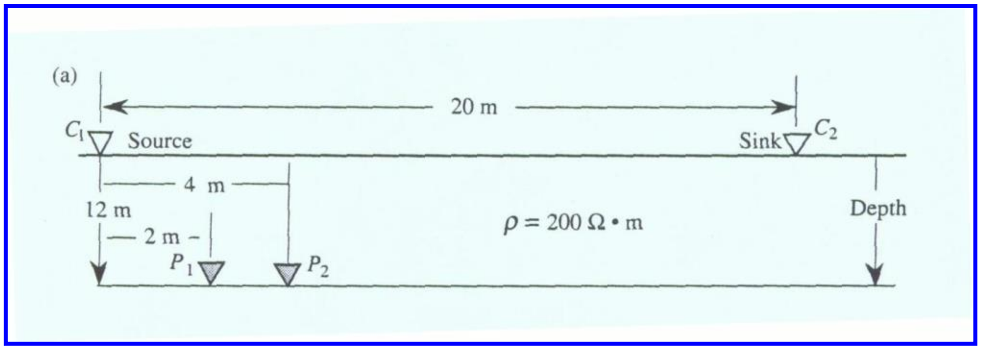
ii) there are situations where, even with increasing velocity in successive layers, a refraction arrival segment may be masked by a deeper higher velocity earlier arriving segment.

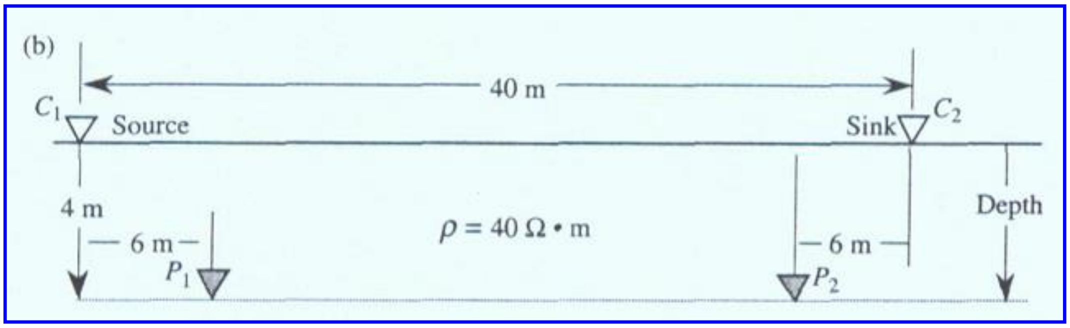
iii) the surface distribution of geophones must extend to distances of several times the anticipated depth of the refractor in order to identify the crossover distance and to determine the slope of the refractor arrival plot.

iv) at the large off-sets required by iii) the arrivals may be very weak and impractically big shot energies may be required

SOLVED PROBLEMS IN ELECTRICAL & EM

1. Determine the potential difference between the two potential electrodes for cases (a) and (b). Assume a current of 0.6 ampere.





(a)

$$V_{P_1} = \frac{i\rho}{2\pi r_1} + \left(-\frac{i\rho}{2\pi r_2}\right) = \frac{i\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{i\rho}{2\pi}\left(\frac{1}{12.17\ \text{m}} - \frac{1}{21.63\ \text{m}}\right)$$

$$V_{P_1} = 0.686\ \text{v}$$

$$V_{P_2} = \frac{i\rho}{2\pi}\left(\frac{1}{12.65\ \text{m}} - \frac{1}{20\ \text{m}}\right)$$

$$V_{P_2} = 0.555\ \text{v}$$

$$V_{P_1} - V_{P_2} = 0.131\ \text{v}$$

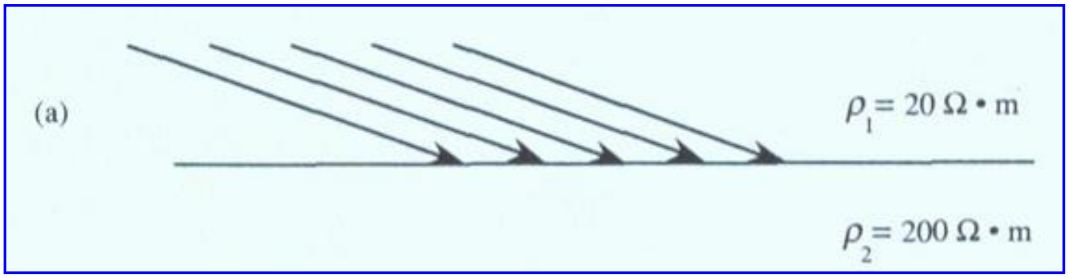
(b)

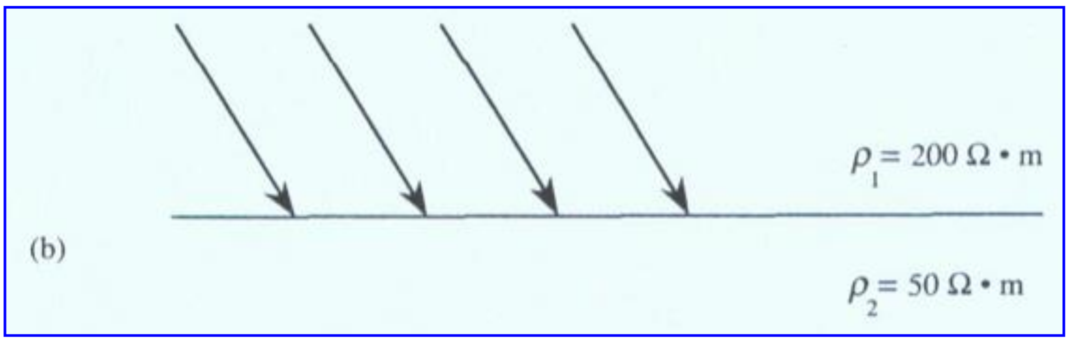
$$V_{P_1} = 0.418\ \text{v}$$

$$V_{P_2} = -0.418\ \text{v}$$

$$V_{P_1} - V_{P_2} = 0.836\ \text{v}$$

2. Construct the current-flow lines beneath the interface in (a) and (b).





(a)

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\rho_2}{\rho_1}$$

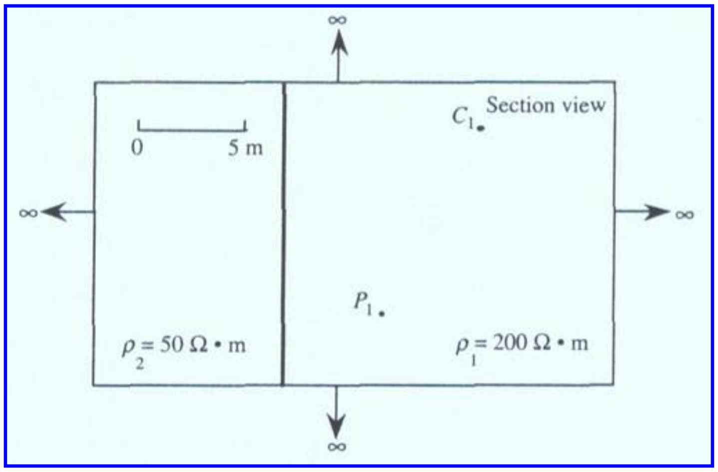
$$\tan\theta_2 = \tan 70° \frac{20\ \text{ohm}\cdot\text{m}}{200\ \text{ohm}\cdot\text{m}}$$

$$\theta_2 = 15.4°$$

(b)

$$\theta_2 = 66.6°$$

3. Calculate the potential at ρ1, due to a current at C, of 0.6 ampere. The material in this section view extends to infinity in all directions. The bold line represents an interface between p1- and p_2-material.



3.

$$V_{P_1} = \frac{i\rho_1}{4\pi r_1} + \frac{ik\rho_1}{4\pi r_2}, \qquad k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} = -0.6$$

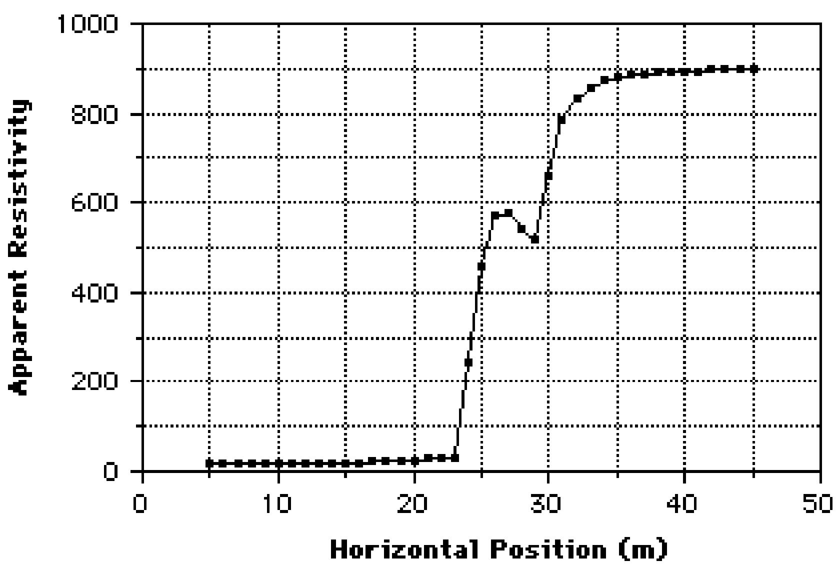
$$V_{P_1} = \frac{0.6\,\text{amp}(200\ \text{ohm}\cdot\text{m})}{4\pi(10\ \text{m})} + \frac{0.6\,\text{amp}(-0.6)(200\ \text{ohm}\cdot\text{m})}{4\pi(16.67\,\text{m})}$$

$$V_{P_1} = 0.611\ \text{v}$$

4. The following data were acquired using a constant-spread, Wenner traverse (a-spacing = 3m). Interpret the data as completely as possible.

Horizontal Position	pa	Horizontal Position (m)	pa	Horizontal Position (m)	Pa
5	20.05	19.00	22.55	33.00	858.94
6	20.06	20.00	25.32	34.00	872.33
7.00	20.07	21.00	28.45	35.00	880.40
8.00	20.08	22.00	27.97	36.00	885.58
9.00	20.10	23.00	27.11	37.00	889.07
10.00	20.12	24.00	242.39	38.00	891.51
11.00	20.15	25.00	460.00	39.00	893.26
12.00	20.19	26.00	572.39	40.00	894.57
13.00	20.24	27.00	580.25	41.00	895.55
14.00	20.32	28.00	541.30	42.00	896.31
15.00	20.44	29.00	519.88	43.00	896.90
16.00	20.61	30.00	660.54	44.00	897.38
17.00	20.91	31.00	785.22	45.00	897.76
18.00	21.45	32.00	834.74		

This data suggests a vertical discontinuity at 25 m horizontal position. Resistivity of the material to the left of the contact is 20 ohm·m and that of the material to the right is 900 ohm·m.



5. The following data were gathered with a Wenner, expanding-spread traverse in an area with thick alluvial deposits at the surface. What is the likely depth to the water table?

Electrode Spacing	pa	Electrode Spacing (m)	pa
0.47	198	6..81	84
0.69	160	10..00	82
1..00	140	14.68	92
1.47	112	21.54	101
2.15	95	31.62	100
3.16	84	46.42	102
4.64	79		

A consistent model shows that layer 1 is 1.3 m, 207 ohm·m; layer 2 is 15.7 m, 77 ohm·m; layer 3 is 107 ohm·m. A variety of input models produce similar results. The water table, therefore, is judged to be at a depth of approximately 1.3 m.

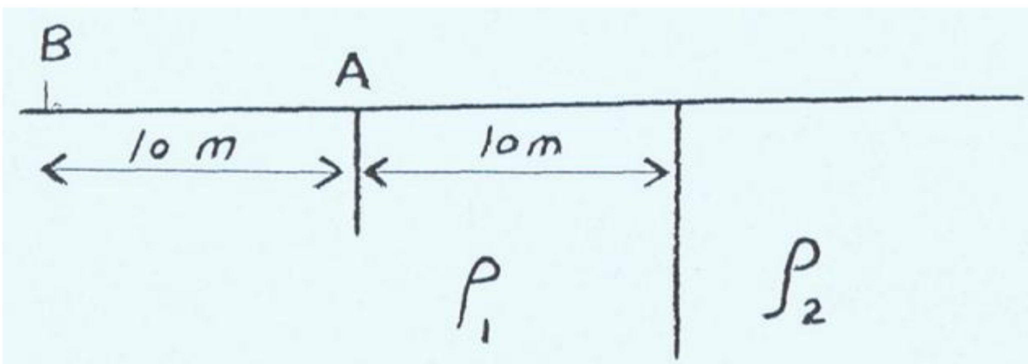
HOMEWORK ASSIGNMENTS IN ELECTRICAL METHODS

1. CONSIDER A VERTICAL CONTACT BETWEEN TWO GEOLOGIC UNITS OF GREATLY DIFFERENT RESISTIVITIES P_1 AND P_2 .

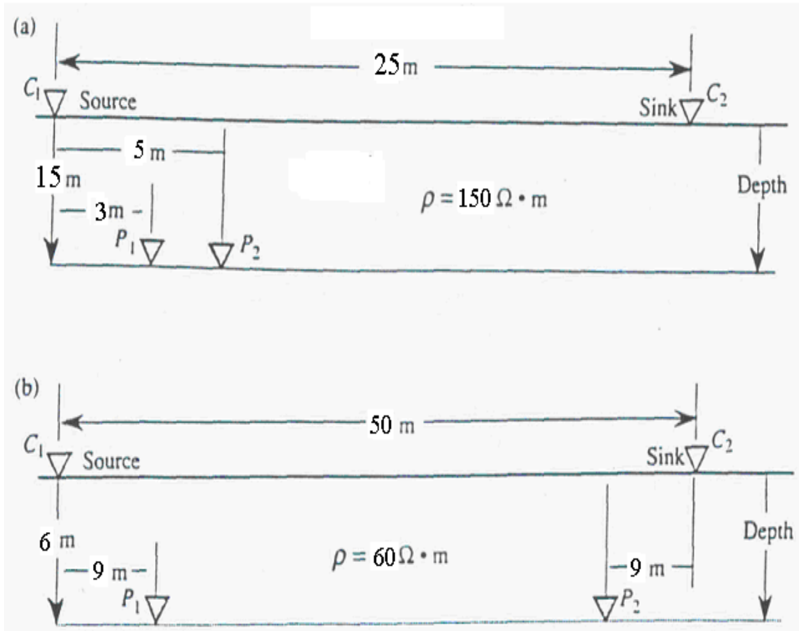
A. CALCULA TE THE DIRECTION THAT ELECTRICAL CURRENT WILL LEAVE THE BOUNDARY INTO P_2 IF IT APPROACHES THE BOUNDARY AT AN ANGLE OF 45° TO THE NORMAL.

B. CALCULATE THE ELECTRICAL POTENTIAL AT POINT A FROM A SINGLE SOURGE OF CURRENT (*+I*) A T POINT B. REMEMBER THAT THE BOUNDARY WILL CREATE A REFLECTED IMAGE OF THE CURRENT SOURCE WITH MAGNITUDE KI. FOR P1 = 100 Ωm AND P_2 = 0.1 Ωm

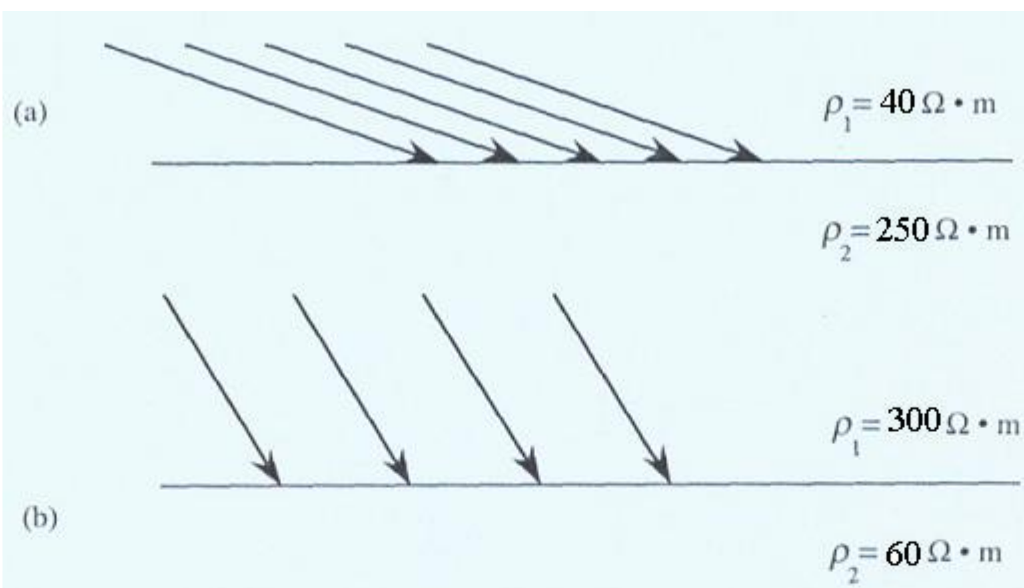
C. REPEAT (A) AND (B) FOR P1 = 100 Ωm AND P_2 = 10000 Ωm



2. Determine the potential difference between the two potential electrodes for cases (a) and (b). Assume a current of 0.5 ampere for (a) and 0.8 ampere for (b).



3. Construct the current-flow lines beneath the interface in (a) and (b).



4 - Interpret the following data, which were obtained with a Schlumberger traverse.

Electrode Spacing (m)	ρ (Ω . m)	Electrode Spacing (m)	ρ (Ω . m)	Electrode Spacing (m)	ρ (Ω . m)
I.00	108	14.68	307	215.44	293
1.47	121	21.54	245	316.23	381
2.15	I4X	31.62	168	464.16	479
3.16	191	46.42	122	681.29	580
4.64	244	68.13	115	1000.0	675
6.81	295	100	162		
10.00	323	146.78	220		

5- The following data were gathered with a Wenner, expanding-spread traverse in an area of thick deltaic sands. Bedrock depths are greater than 30 m. What is your best estimate of the depth to the water table in this area?

Electrode Spacing (m)	ρ (Ω . m)	Electrode Spacing (m)	ρ (Ω . m)
0.47	2590	6.81	8753
0.69	3288	10.00	7630
1.00	4421	14.68	4805
1.47	5198	21.54	2160
2.15	6055	31.62	995
3.16	6686	46.42	584
4.64	7782		

6. The following data were gathered with a Wenner. expanding-spread traverse in an area of dune sands underlain by lake clays which in turn are underlain by Triassic sedimentary rocks. Estimate a value for clay thickness.

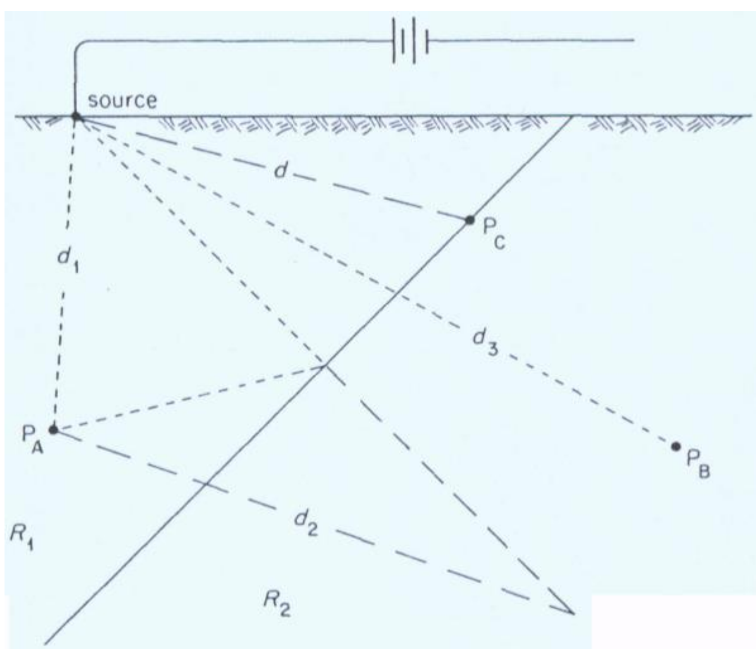
Electrode Spacing (m)	ρ (Ω . m)	Electrode Spacing (m)	ρ (Ω . m)
0.69	1298.90	10.00	360.67
1.00	1398.06	14.68	240. 1 1
1.47	1306.98	21.54	191.51
2.15	1153.02	31.62	153.81
3.16	925.27	46.42	116.38
4.64	762.40	68.13	98.03
6.81	554.13	100.00	86.08

7. Interpret the following data, which were obtained with a Wenner traverse.

Electrode Spacing (m)	p_a (Ω • m)	Electrode Spacing (m)	p_a (Ω • m)	Electrode Spacing (m)	p_a (Ω • m)
1.00	984	14.68	72	215.44	327
1.47	955	21.54	53	316.23	432
2.15	883	31.62	63	464.16	554
3.16	742	46.42	87	681.29	686
4.64	533	68.13	124	1000.00	818
6.81	311	100.00	174		
10.00	150	146.78	241		

8. Assume a homogeneous medium of resistivity 120 ohm-m. Using the wenner electrode system with a 60-m spacing, assume a current of 0.628 ampere. What is the measured potential difference? What will be the potential difference if we place the sink (negative-current electrode) at infinity?

9. Suppose that the potential difference is measured with an electrode system for which one of the current electrodes and one of the potential electrodes are at infinity. Using the Figure below and a current of 0.5 ampere, compute the potential difference between the electrodes at P_A and infinity for $d_1 = 50$ m, $d_2 = 100$ m, $R_1 = 30$ ohm-m, $R_2 = 350$ ohm-m.



10. Plot resistivity data as a function of electrode spacing, and determine the particular electrode spacing corresponding to the inflection point for the data given in Exercise 9. Compare the electrode spacing at the inflection point with the depth of the boundary from Exercise 9.

11. If the Schlumberger electrode system with AB/2 : MN/2 = 5. is used to conduct the resistivity survey explained in Exercise 9, what will the potential readings be? Use resistivity values found in Exercise 9 to compute potential differences for each of the electrode spacings AB/2 = 1, 2, 4, 6, 8, 10, 15, 20, 25, 30, 40, 50 m while a constant current of 0.250 ampere is applied.

12. Suppose that an electrical resistivity survey was done using an expanding Wenner electrode-configuration. The current of 0.25 ampere was kept the same for all the readings. Potential differences measured with different electrode spacings are given in the following table. Interpretations of these measurements indicate that a layer of resistivity R_1 lies above another layer of resistivity $R_{.2}$. Determine the depth of the boundary between these two layers. Estimate the resistivities of the layers.

ELECTRODE SPACING (m)	**POTENTIAL DIFFERENCE (v)**
1	0.8
2	0.42
4	0.28
6	0.155
8	0.125
10	0.120
15	0.105
20	0.10
25	0.098
30	0.086
40	0.076
50	0.064

13. For purposes of an IP survey, resistivity values are determined from both direct and alternating current using the same electrode arrangement. If the resistivities for direct and alternating current are R_{dc} = 50 ohm-m and R_{ac} = 40 ohm-m, respectively, what will the frequency effect and the metal factor values be.

14. Assume- that a telluric current survey is to be carried out to outline large-scale features of a sedimentary basin 5 km deep. A resistivity of 50 ohm-m is supposed to represent the sedimentary section. What is the maximum frequency of the telluric current that will penetrate below the basin?

15. Suppose That a magnetotelluric survey indicates an apparent resistivity of 5 ohm-m at a frequency of 1 Hz, What is the thickness of the layer?

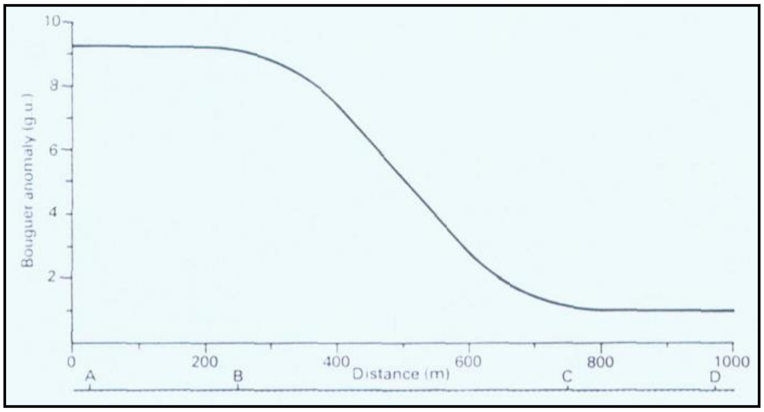
16. Using the method of electrical images, drive therelationship between apparent resistivity, electrode spacing, layer thicknesses and resistivities for a VES performed with a Schlumberger spread over a single horizontal interface between media with resistivities ρ_1 and p_2.

17. Calculate the variation in apparent resistivity along a HEP profile at right angles to a vertically faulted contact between sandstone and limestone, with apparent resistivities of 50 ohm m and 600 ohm m. respectively, for a Wenner configuration. What would be the effect on the profiles if the contact dipped at a shallower angle.

18. Why are the electrical methods of exploration particularly suited to hydrogeological investigations? Describe other geophysical methods which could be used in this context, stating the reasons why they are applicable.

19. Let *A* and B represent two different geologic sections, and let H_1 and H_2 represent the thicknesses of the first and second layers in a three-layer sequence. It is well known that one type of equivalence occurs for a three-layer case when $p_1 < p_2 > p_3$. $p_{1A} = p_{1B}$, $p_{3A} = p_{3B}$, $H_{1A} = H_{1B}$, and $P_{2A}. H_{2A} = P_{2B}. H_{2B}$. Demonstrate that this is true. Does equivalence exist if $p_1 > p_2 < p_3$?. Explain.

20. At locations A, B, C , D along the gravity profile shown below, VES were performed with a Wenner array with the spread laid perpendicular to the profile.



It was found that the sounding curves, were similar for locations A ,and B and for C and D. A borehole close to A penetrated 3m of drift, 42 m of limestone and bottomed in sandstone. Downhole geophysical surveys provided the following values of density (ρ_D) and resistivity (ρ_R) for the lithologies encountered.

Unit	ρ_R (Ω m)	ρ_D (Mg m^{-3})
Drift	40	2.00
Limestone	2000	2.75
Sandstone	200	2.40

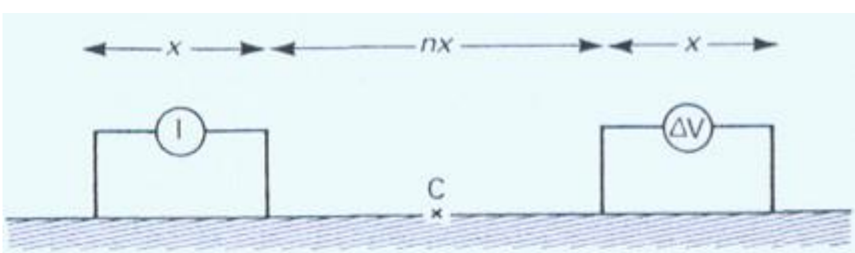
A seismic refraction line near to D revealed 15m of drift, although the nature of the underlying basement could not be assessed from the seismic velocity.

(a) Interpret the geophysical data so as to provide a geological section along the profile.
(b) What further techniques might be used to confirm your interpretation?
(c) If a HEP were to be performed along the profile, select, giving reasons, a suitable electrode spacing to map the basement. Sketch the expected form of the HEP for both longitudinal and transverse traverses.

21. The following table represents the results of a frequency domain IP survey of a Precambrian shield area. A dipole-dipole array was used with the separation (x) of both the current electrodes and the potential electrodes kept constant at 60m. n refers to the number of separations between the current and potential electrode pairs and c to the distance of the center of the array from the origin of the profile, where the results are plotted (Figure below). Measurements were taken using direct current and an alternating current of 10Hz. These provided the apparent resistivities ρ_{dc} and ρ_{ac} respectively,

(a) For each measurement point, calculate the percentage frequency effect (PFE) and metal factor parameter (MF).

	$n = 1$		$n = 2$		$n = 3$		$n = 4$	
c (m)	ρ_{dc} (Ω m)	ρ_{ac} (Ω m)	ρ_{dc} (Ω m)	ρ_{ac} (Ω m)	ρ_{dc} (Ω m)	ρ_{ac} (Ω m)	ρ_{dc} (Ω m)	ρ_{ac} (Ω m)
0	49.8	49.6			101.5	100.9		
30			72.8	72.4			99.6	98.5
60	46.0	45.8			86.2	85.2		
90			61.3	60.6			90.0	86.1
120	42.1	41.7			72.8	70.1		
150			55.5	54.4			57.5	53.5
180	44.0	43.5			49.8	46.6		
210			53.6	51.1			47.9	44.0
240	42.1	41.8			44.0	41.4		
270			65.1	64.1			47.9	44.9
300	49.8	49.6			95.8	91.7		
330			82.3	81.3			132.1	129.4
360	51.7	51.3			114.9	114.1		
390			86.2	85.9			164.7	164.0
420	49.8	49.6			120.7	120.1		
450			78.5	78.0			170.4	169.7



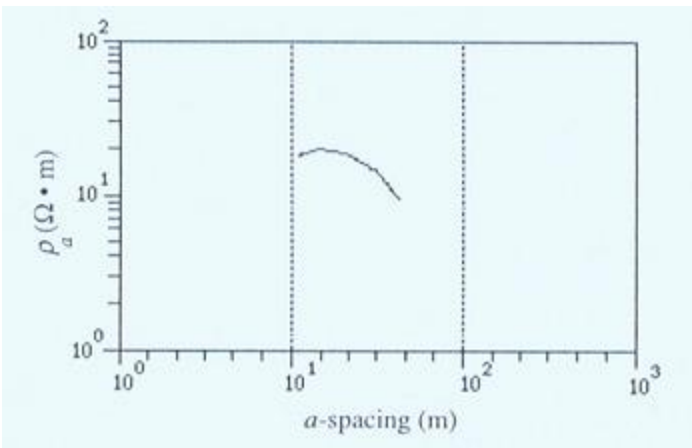
The dipole-dipole electrode configuration.

(b) For both the PFF, and MF plot four profiles for n = 1,2 , 3 and 4.

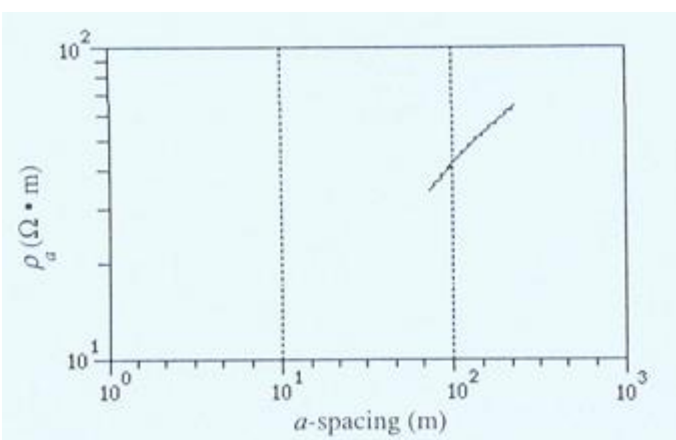
(c) The area is covered by highly-conductive glacial deposits 30-60m thick, It is possible that massive sulphide mineralization is present within the bedrock. Bearing this information in mind, comment upon and interpret the profiles.

22. For each of the following subsurface models sketch an appropriate apparent resistivity curve on the designated graph. The general shape of the curve is what is important. Base your curves on what you know about current penetration, current density, and measured resistivities.

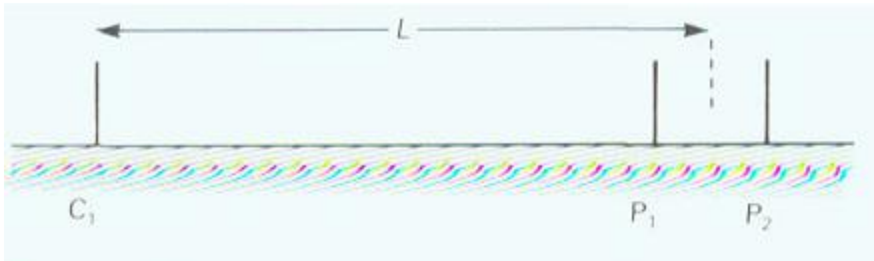
(a) Layer	Thickness (m)	Resistivity (Ω.m)
1	1	100
2	5	10
3	5	100
	4	1
	Infinite	



(b) Layer	Thickness (m)	Resistivity (Ω.m)
1	10	100
2	10	10
3	Infinite	100

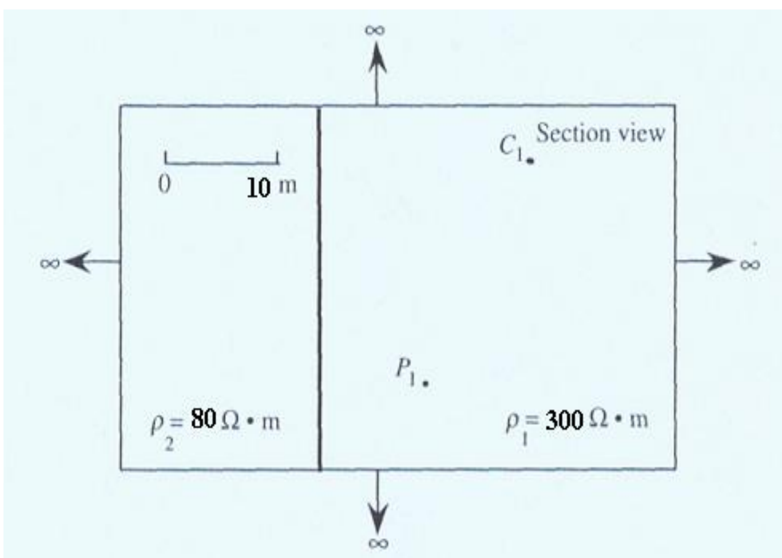


23. The figure below shows a half-Schlumberger resistivity array in which the second current electrode is situated at a great distance from the other electrodes. Derive an expression for the apparent resistivity of this array in terms of the electrode spacings and the measured resistance.



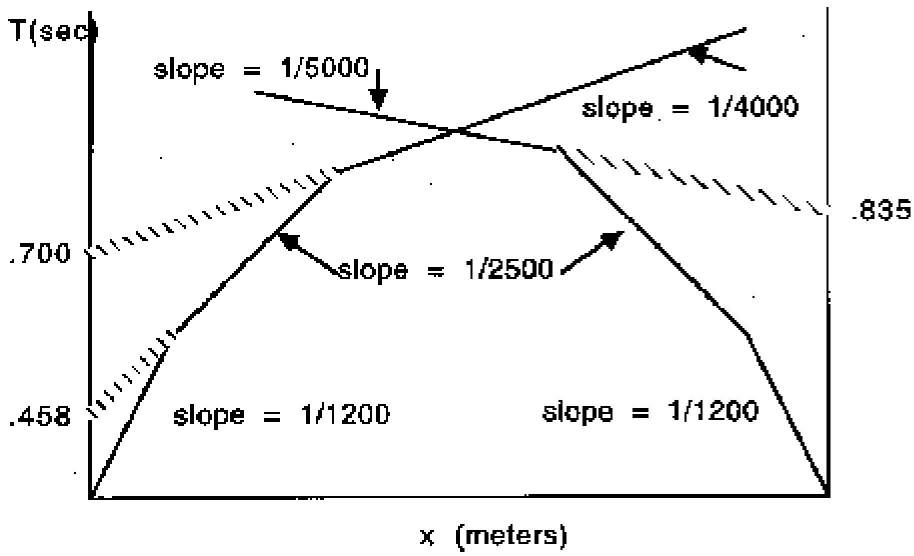
The half-Schlumberger electrode configuration.

24. Calculate the potential at P_1 due to a current at C_1 of 0.6 ampere. The material in this section view extends to infinity in all directions. The bold line represents an interface between p_1 and p_2 –material.

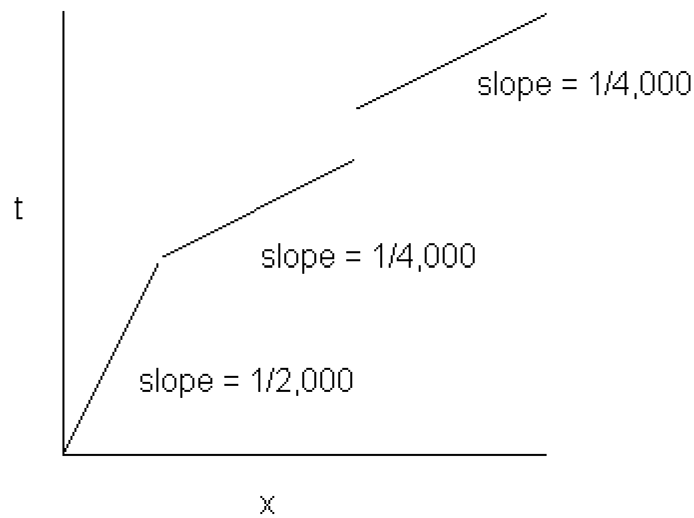


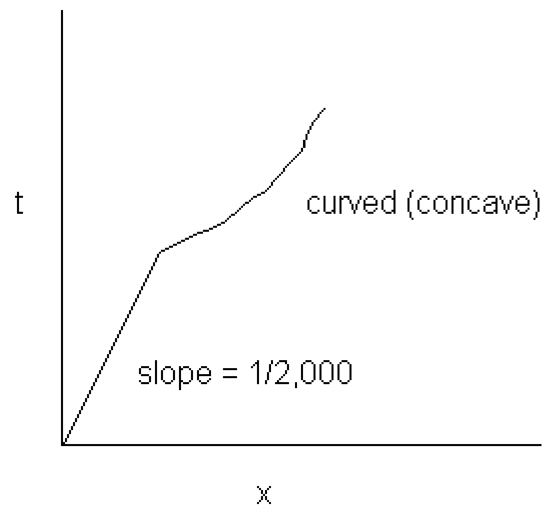
PROBLEM SETS IN GEOPHYSICAL EXPL.

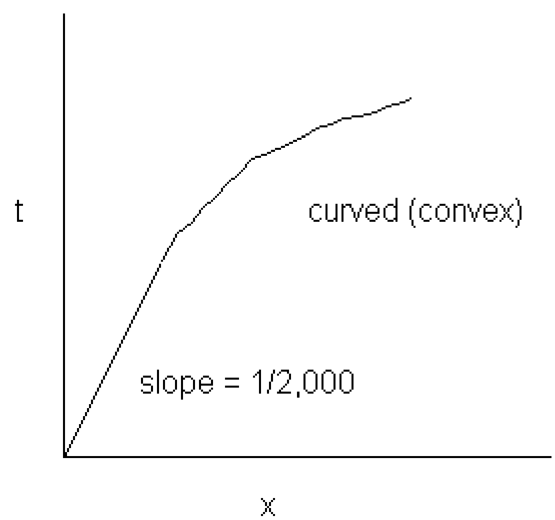
1. Given the reversed refraction observations (travel time vs. distance curves) shown below, calculate the velocities and depths to the interfaces. Calculate the dip angles of the interfaces.



2. Given the following schematic travel-time curves, describe a subsurface structure and/or velocity changes that may explain them.







3. A rock sample is taken to the lab and is subjected to a uniaxial stress (that is, it is stressed in only one direction with the remaining directions free). As a result of the stress, the length of the sample increases by 3% and the width decreases by 1%. What is the ratio of the P wave velocity to the S wave velocity in this sample?

4. Two seismic reflection profiles are shot, the first along a north-south line and the second along a line oriented 45° east of north. The first profile reveals an interface dipping to the north at 20°, while the second reveals an interface dipping to the northeast at 35°. What is the true strike and dip of the interface?.

5. A material has a shear modulus of 8.8×10^9 Pa, a bulk modulus of 2.35×10^{10} Pa, a density of 2200 kg/m^3 and a quality factor Q = 100.

a) What are the P-wave velocity and S-wave velocity of the medium, in units of km/s ?
b) If a P-wave of frequency 10 Hz has a displacement amplitude of 1 µm at a distance of 10 m from the source, what would be the wave amplitude at 100 m?

6. Near-surface fresh water in a Lake Superior has been observed to have a P-wave velocity of 1435 meters/sec. Estimate its bulk modulus, assuming it is pure water.
2. A laboratory has determined that the Gabbro has the following properties:
Bulk modulus= 0.952130 X 1012 dyne/cm2
Shear modulus=0.403425 X 1012 dyne/cm2
Density=2.931 gm/cc

Determine the (a) shear wave velocity, the (b) compressional wave velocity, and (c) Poisson's ratio for this rock.

7. While attempting to put in a water-well near Longville, Minnesota, a driller encountered fresh granite at "about 50 feet". Because the glacial till, which largely consists of unconsolidated clay and silt, is usually thicker in this region, the driller first suspected that he had hit a boulder. Upon drilling a "few more feet", however, the granite persisted, leading the driller to fear that he had indeed hit bedrock, and that further drilling might result in an expensive and potentially unproductive water well.
A seismic refraction survey that used a sledge hammer for an energy source was conducted at the site to investigate the depth to bedrock. The observed first breaks were as follows:

Dist. from source (ft.)	**t first break (milliseconds)**
5	5.19
10	9.74
30	21.8
60	34.9
90	40.7
120	46.7
150	51.1
180	54.3
210	59.3
240	65.5
270	72.2
300	75.2
330	79.8

A reversed profile (not given) was essentially identical, and, therefore, horizontal layering can be assumed. Plot the travel time graph (x-horizontal, t-vertical) and assuming a two-layer model, estimate v1, v2 and h1. Assuming a velocity of 18,000 feet/second for fresh granite,
do you see any evidence of granite bedrock in the profile? If so, what is the depth? If not, what is the minimum depth that granite could occur?
Do you think that the driller hit bedrock or a very large boulder?

8. In Todd County, Minnesota a seismic refraction survey was conducted along an abandoned railroad grade about 2.5 miles southeast of the town of Osakis. The railroad grade is known to be resting on Pleistocene glacial deposits. A 12 channel system was used with an sledge hammer for an energy source, and the following first break times were picked from the traces.

Distance (meters)	**First break time (milliseconds)**
5	14
15	23
25	30
35	36.75
45	42
55	50
65	54.5
75	61
85	65.5
95	66.5
105	70
115	73.25

A reversed profile yielded essentially identical results, so we can assume horizontal layering. It is assumed that the first and second layers will represent the grade fill and the glacial deposits, respectively.

Plot the travel time relationships and estimate the velocities of the first two layers (in meter/second). What is the thickness of the grade fill?
Based on your velocity estimate for the second layer, do you think the glacial deposits are saturated or unsaturated (below or above the water table)?
Do you have any evidence of bedrock (ie a third layer) below the glacial deposits? If so, what is its velocity (in meters/second) and depth?

9. FOR PURPOSE OF IP SURVEY, RESISTIVITY VALUES ARE DETERMINED FROM BOTH DIRECT AND ALTERNATING CURRENT USING THE SAME ELECTRODE ARRANGEMENT. IF THE RESISTIVITY FOR DIRECT AND ALTERNATING CURRENT ARE R_{dc} = 50 OHM.M AND R_{ac} = 40 OHM.M, RESPECTIVELY. WHAT WILL THE FREQUENCY EFFECT AND THE METAL FACTOR VALUES BE.

10. DESCRIBE THE APPLICABILITIES OF SEIMIC AND ELECTRICAL (WENNER, SCHLUMBERGER, DIPOLE-DIPOLE) , AND GROUND PENETRATING RADAR (GPR) METHODS FOR INVESTIGATING THE FOLLOWING TARGETS :

A. SOLUTION CAVITIES AND FRACTURES IN LIMESTONE FORMATION.

B. SPHALERITE AND GALENA (LEAD AND ZINC ORES) MASSIVE SULFIDE DEPOSITS IN A DOLOMITE / MARBLE FORMATION.

C. MAPPING OF SALWATER / FRESHWATER INTERFACE ; SATURATED AND UNSATURATED ZONES IN COASTAL AREAS.

D. ARCHAEOLOGICAL APPLICATIONS

11. CALCULATE ELEVATION AND WEATHERING CORRECTIONS FOR THE THREE GEOPHONE LOCATIONS IN THE ACCOMPANYING SKETCH.

12. SUPPOSE THAT A REVERSED REFRACTION SURVEY (USING SHOTS A AND B) INDICATED VELOCITIES V1 = 1500 M/SEC. AND V2 = 2500 M/SEC. FROM SHOT A AND VELOCITIES V1 = 1500 M/SEC AND V2 = 3250 M/SEC. FROM SHOT B.

FIND THE DIP OF REFRACTOR. WHAT WOULD BE THE CHANGES IN VELOCITIES IF THE FEFRACTOR HAD A SLOPE 10 DEGREES LARGER THAN THE ONE YOU COMPUTED.

13. SUPPOSE THAT THE POTENTIAL DIFFERENCE IS MEASURED WITH AN ELECTRODE SYSTEM FOR WHICH ONE OF THE CURRENT ELECTRODES AND ONE FOR THE POTENTIAL ELECTRODES ARE AT INFINITY. USING THE FIGURE BELOW, AND A CURRENT OF 0.5 AMPERE.

COMPUTE THE POTENTIAL DIFFERENCE BETWEEN THE ELECTRODES AT P_A AND INFINITY FOR d1 = 50 m, d2 = 100 m, R1 = 30 Ohm-m, R2 = 350 Ohm-m.

.14. A SET OF SEISMIC REFRACTIN FIELD DATA WAS COLLECTED FOR BOTH FORWARD AND REVERSE SHOTS AS GIVEN BELOW :

DISTANCE (FT)	**ELEVATION (FT)**	**ARRIVAL TIMES (MSEC)**	
		SHOT A	**SHOT B**
0 (SHOT A)	105	0	---------
100	103	48	274
200	101	93	252
300	95	136	237
400	90	139	221
500	85	151	211
600	110	176	221
700	110	202	212
800	105	221	204
900	100	238	179
1000	105	237	136
1200	110	271	47
1300 (SHOT B)	95	-----	0

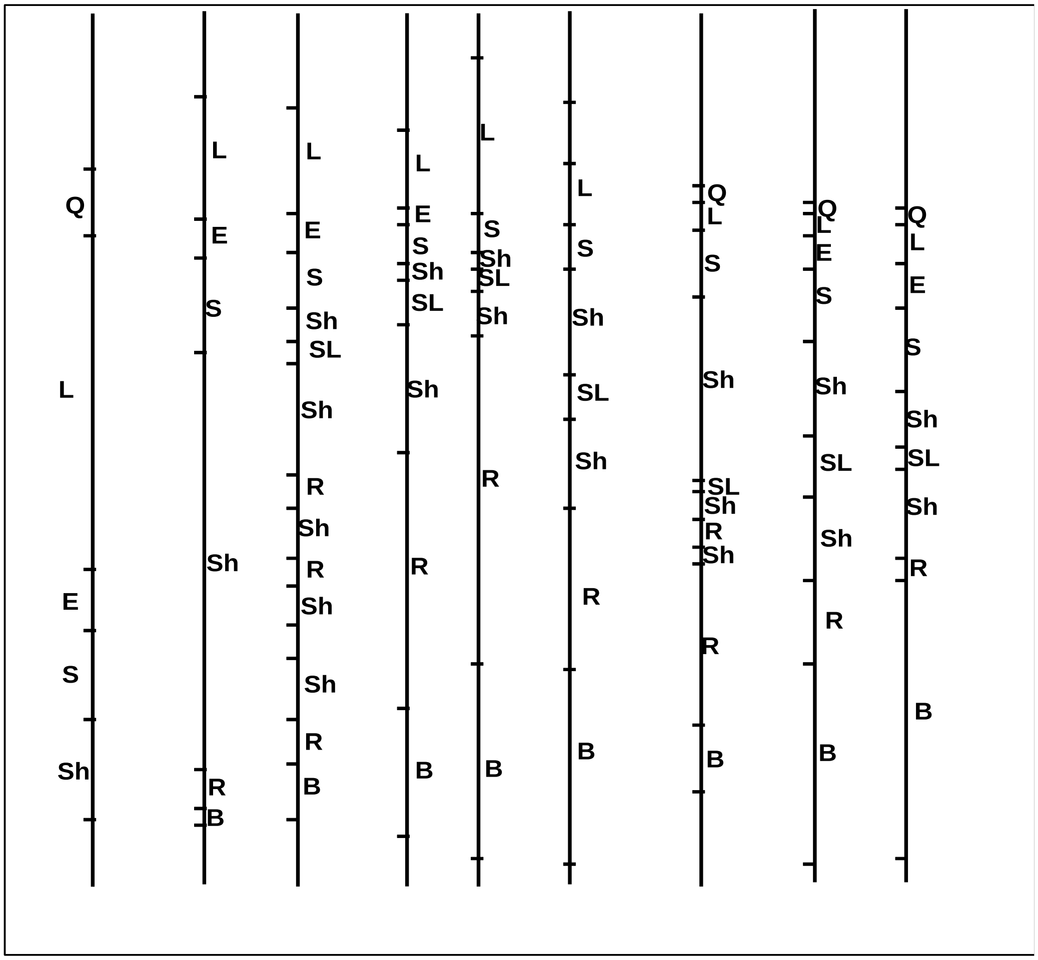
A. PLOT T-X CURVE (WITHOUT ANY CORRECTION). ASSUMING A SLOPING LAYER, OBTAIN AND PLOT THE SUBSURFACE STRUCTURE.

B. APPLY AN ELEVATION CORRECTION TO EACH STATION FOR THE ENTIRE RECORDS. RE-INTERPRET THE T-X CURVE AND PLOT THE SUBSURFACE STRUCTURE.

C. APPLY BARTHELEMES' DELAY TIME METHOD FOR EACH SHOT POINT A AND B. DRAW THE FINAL CROSS-SECTION OF THE SUBSURFACE ALONG WITH THE SURFACE TOPOGRAPHY.

15. DETERMINE THE SUITABLE OIL ACCUMULATION FROM THE CROSS-SECTION

Exercise No.( )



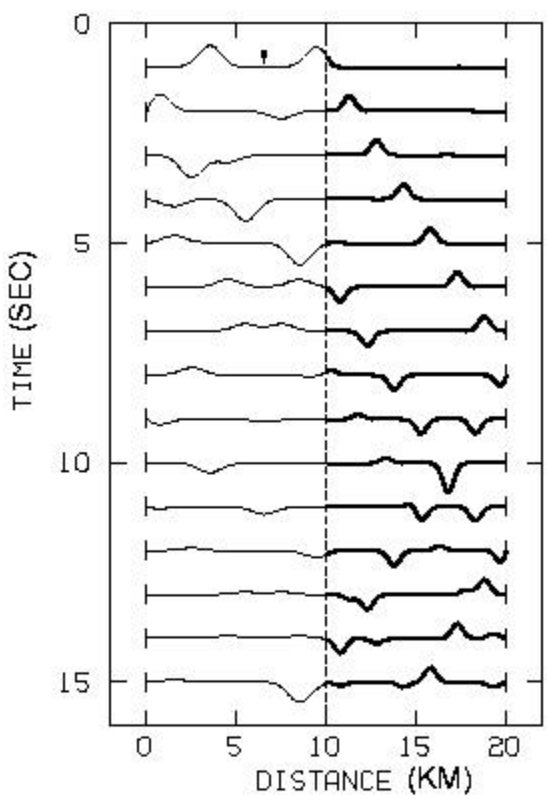
Complete the given geologic cross-section and determine the suitable location of oil accumulation.

Q = Quaternary, **L** = hard Eocene limestone,
E = Eocene evaporite , **S** = Paleocene sandstone,
Sh = Upper Cretaceous shale, **SL** = sand lens,
R = coral reefs, **B** = Basment

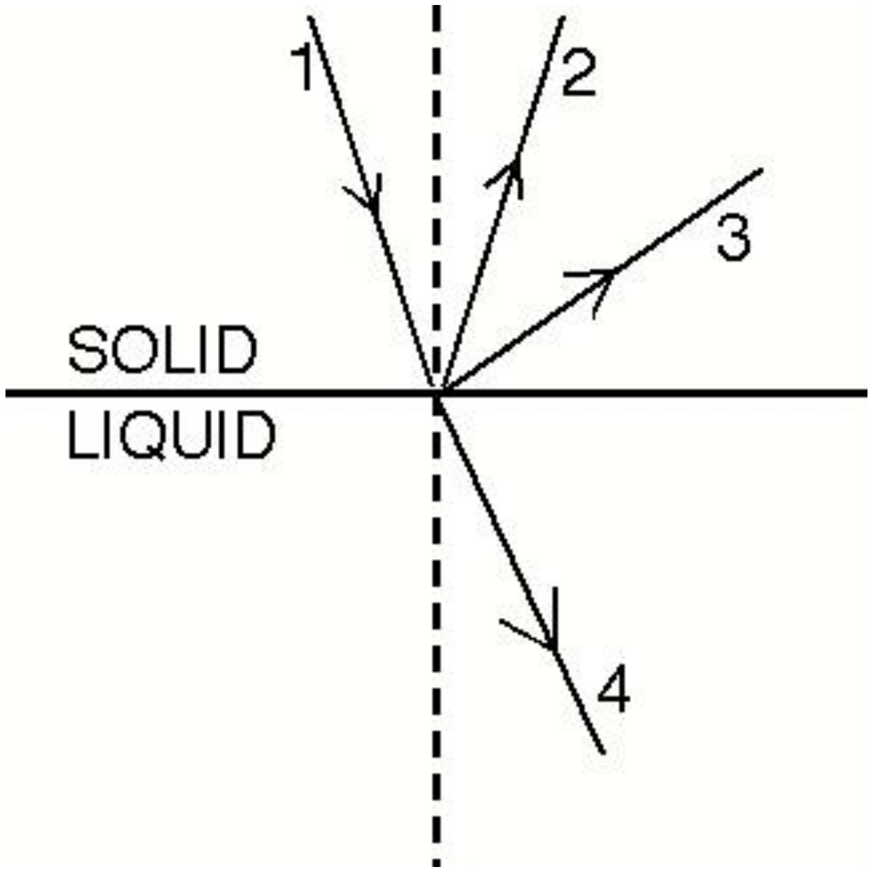
SHOWN.

16. The figure below shows waves travelling on a string whose right and left sides have a different velocity and density.

a. Use the positions of the waves at successive times to find the velocity on either side.
b. Measure the amplitude of the wave incident on the junction (dashed line) at time 1, and the amplitudes of the reflected and transmitted waves at time 2. Find the reflection and transmission coefficients from the ratio of these amplitudes.
c. Derive a relationship, using the expression for the transmission coefficient, for the ratio between the densities of the two media. Which medium is the more dense?
d. If the density of the right hand medium is $\rho_2 = 4 \text{ g} / \text{cm}^3$, determine the density of the left hand medium.
e. Do these waves more nearly represent P-waves or S-waves? Why?



17. Identify the four wave types shown (P or S). Assuming the P velocity in the solid is 13.5 km/s, find the P and S velocities in both media. (Hint: Use a protractor to measure the angles).



18. For a layer of thickness h, composed of material with velocity V_0 above a half-space which consists of material with a velocity V_1, as shown below,

a. Express the travel time $T_{Direct}(x)$ of the direct wave to the receiver as a function of the source-receiver distance *x*.

b. Show that the travel time of the reflected wave T_{Refl} can be represented as, $T_{Refl}(x) = \frac{\sqrt{x^2 + 4h^2}}{v_0}$ To test this, determine the travel time for the reflection at zero distance (zero offset).

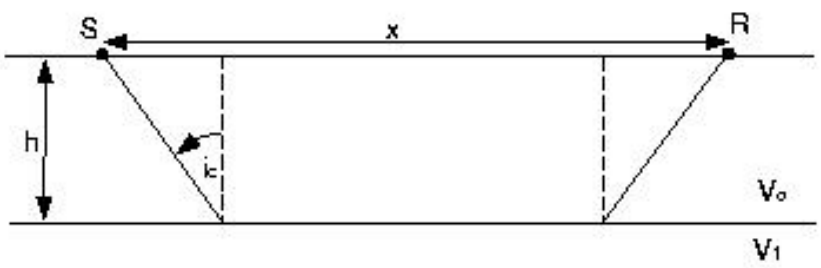
c. Express the critical angle ic as a function of the velocities V_0 and V_1 .

d. Show that the travel time for a critically refracted (or head) wave at a distance x is; $T_{Refl}(x) = \frac{x}{v_1} + 2h\frac{\sqrt{(v_1^2 - v_o^2)}}{v_1 v_o}$

e. Derive an expression for the minimum distance at which the refracted wave may be observed in terms of the layer thickness *h* and the velocities V_0 and V_1 .

f. Show that the crossover distance at which the direct and refracted wave arrivals arrive at the same time is given by; $X_d = 2h\sqrt{\frac{v_1 + v_o}{v_1 - v_o}}$.

g. Plot $T_{Direct}(x)$, and $T_{Refl}(x)$ as a function of distance for h = 33 km, V_0 = 6 km/s, V_1= 8 km/s.



مصطلحات الإستكشاف الكهربائي والكهرومغناطيسي

الترجمة العربية	المصطلح
جهد ذاتي	Self Potential
مقياس الجهد الكهربائي	Galvanometer
مقاومية ظاهرية	Apparent Resistivity
خط متساوي الجهد	Equipotential line
مبدأ التعادل	Principle of Equivalence
مبدأ الغطس	Principle of Suppression
التوصيلية	Conductivity
الجس الكهربائي العمودي	Vertical Electrical Sounding
المقطع الكهربائي الأفقي	Horizontal Electrical Profiling
ثنائي القطب	Dipole
ترتيب شلمبرجير	Schlumberger Arrangement
الإستقطابية المستحثة	Induced Polarization
الطرق الخامدة	Passive Methods
التوصيل الكهربائي	Electrical Conduction
قانون أرشي	Archie's Law
معامل الإنعكاسية الكهربائية	Electrical Reflection Coefficient
رادار الإختراق الأرضي	Ground Penetrating Radar (GPR)
المسح الكهرومغناطيسي الجوي	Airborne Electromagnetic
التردد المنخفض جدا	Very Low Frequency (VLF)

مصطلحات الإستكشاف الجاذبي والمغناطيسي

Absolute gravity	جاذبية مطلقة
Magnetic Field Strength	قوة المجال المغناطيسي
Magnetic Induction	الحث المغناطيسي
Centrifugal Force	قوه طرد مركزية
Intensity of Magnetization	شدة التمغنط
Magnetic Declination	انحراف مغناطيسي
Diamagnetic	ضعيف النفاذية المغناطيسية
Diurnal Correction	تصحيح يومي
Elevation Correction	تصحيح الارتفاع
Equator	خط الاستواء
Magnetic Permeability	النفاذية المغناطيسية
Ferro magnetic	مغناطيس حديدي
Gravitational Acceleration	التسارع الجاذبي
Gravity Anomaly	شاذة الجاذبية
Magnetic Inclination	الميل المغناطيسي
Isostatic Correction	تصحيح ايزوستاتي
Lunar Variations	تغيرات قمرية
Magnetic Moment	العزم المغناطيسي
Magnetic Storms	عواصف مغناطيسية
Magnetometer	جهاز قياس المغناطيسية
Magnetic Susceptibility	قابلية مغناطيسية (التأثرية المغناطيسية)
Observed Gravity	جاذبية مقاسة
Paleomagnetism	مغناطيسية قديمة
Residual Magnetism	مغناطيسية متخلفة
Remanent Magnetism	مغناطيسية متبقية
Secular Variations	تغيرات متناهية البطء
Geoid	الجيوئد (سطح متساوي الجهد)
Gravimeter	جهاز قياس الجاذبية
Bouguer Anomaly	شاذة بوجير
Latitude Correction	تصحيح خط العرض

مصطلحات الإستكشاف السيزمي

المصطلح	الترجمة العربية
Elasticity	المرونة
Stress System	نظام الإجهاد
Poisson's ratio	معامل بوايسون
Tangential Stress	الإجهاد المماسي
Transverse Stress	الإجهاد المستعرض
Transverse Strain	التشوه (الانفعال) المستعرض
Normal Stress	الإجهاد العمودي
Rigidity Modulus	معامل الصلابة
Shear Modulus	معامل القص
Hooke's Law	قانون هوك
Elastic Limit	حد المرونة
Plastic Point	نقطة اللدونة
Anelastic Materials	المواد اللامرنة
Shear resistance	مقاومة القص
Young's modulus	معامل يونج
Compressibility	الانضغاطية
Dilatation	تمدد حجمي
Wave Propagation	الانتشار الموجي
Body Waves	الموجات الباطنية
Surface Waves	الموجات السطحية
Longitudinal Waves	الموجات الطولية
Primary Waves	الموجات الأولية
Compressional Waves	الموجات التضاغطية
Shear Waves	موجات القص
Transverse Waves	موجات مستعرضة
Secondary Waves	موجات ثانوية
Birch's Law	قانون بيرش
Seismic Velocities	سرع سيزميه
Rayleigh Waves (LR)	موجات رايلي
Love Waves (LQ)	موجات لوف
Dispersion	تشتت
Amplitude	سعة الموجة
Wavelength	طول الموجة
Frequency	تردد

Seismic Refraction	الانكسار السيزمي
Seismic Reflection	الانعكاس السيزمي
Critical Distance	المسافة الحرجة
Thickness	سماكة
Depth	عمق
Seismic Source	مصدر سيزمي
Transmitter	مرسل
Receiver	مستقبل
Geophones	سماعات أرضية
Fermat's Principle	مبدأ فيرمات
Huygen's Principle	مبدأ هايجن
Reflection Coefficient (R_c)	معامل الانعكاس
Transmission Coefficient (T_c)	معامل الاختراق
Acoustic Impedance	العائق الصوتي
Wavefront	مقدمة الموجة
Raypath	مسار الموجة
Snell's Law	قانون سنيل
Critical Refraction	الانكسار الحرج
Low- Velocity – Layer	طبقة منخفضة السرعة
Hidden Layer	طبقة مختبئة
Blind Layer	طبق عمياء
Thin Layer	طبقة رقيقة
Diffraction	الحيود
Delay Time	زمن التأخير
Dipping Layers	طبقات مائلة
Green Equation	معادلة جرين
Dynamic Correction	التصحيح الديناميكي
Multiple Reflection	انعكاس متعدد
Time- Average Equation	معادلة معدل الزمن
Faust Equation	معادلة فوست
Apparent Velocity	سرعة ظاهرية
Average Velocity (VA)	معدل سرعة
Interval Velocity (VI)	سرعة بينية
Root Mean Square Velocity	سرعة تربيع متوسط الجذر
Dix Equation	معادلة ديكس
Data Processing	معالجة المعلومات
Cross Over Distance	مسافة العبور
Seismic Attenuation	تعتيم سيزمي